MMME2046 Dynamics and Control: Control Lecture 2

Overcoming Non-Linearity

Position Control Systems

Brilliant Idea no. 2: Hydraulic position control

Lecture Objectives:

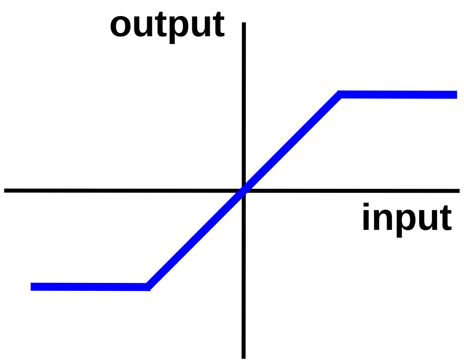
- Discuss non-linearity and linearisation
- Introduce transient and steady-state responses
- Introduce position control systems

Non-Linearity

- Sometimes, components of a system will not reduce to a simple linear relationship
  - Superposition does not apply
  - Laplace Transforms are not valid

Non-Linearity

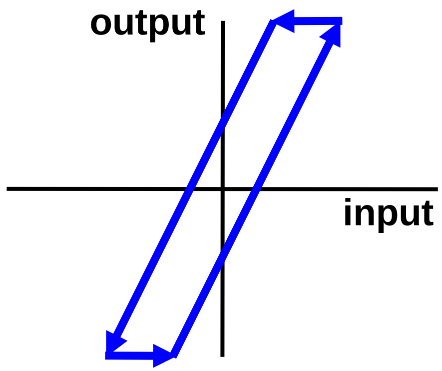
- Saturation
  - Applies to amplifiers, actuators, power supplies, batteries …

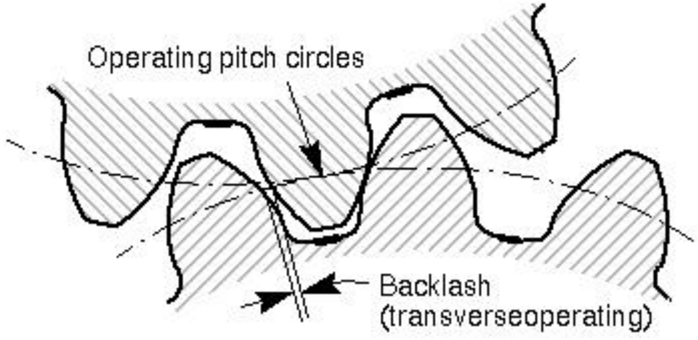


  - There is a physical limit on the maximum output, or rate of change

Non-Linearity

- Backlash (particularly in gears)
  - Particularly in older or cheaper systems (wear, tolerance)

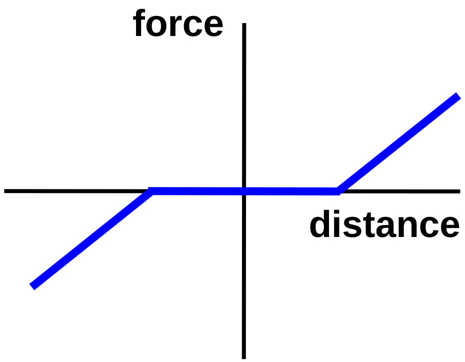




  - Change in direction results in change of input/output relationship

Non-Linearity

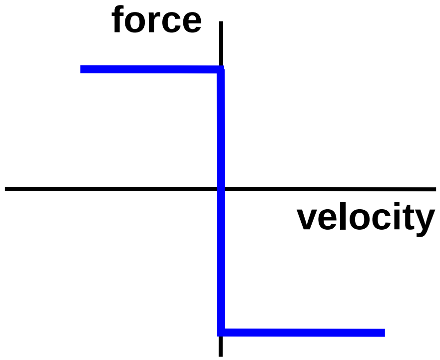
- Clearance Effects
  – Similar to backlash – actuator moves before load is engaged (taking up the slack)



  – Small displacement for zero force

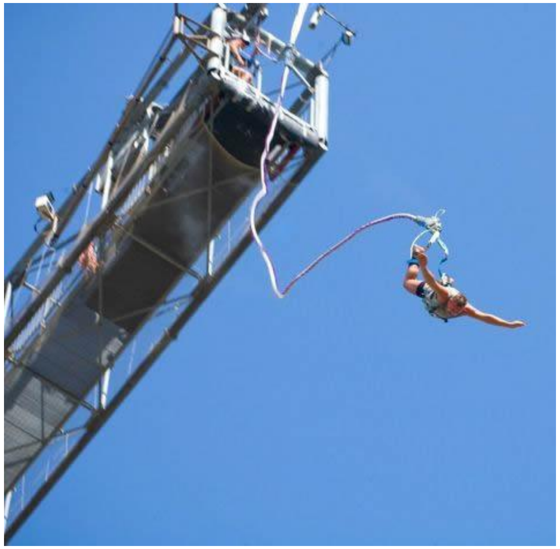
Non-Linearity

- Coulomb friction
  - Constant force opposes movement

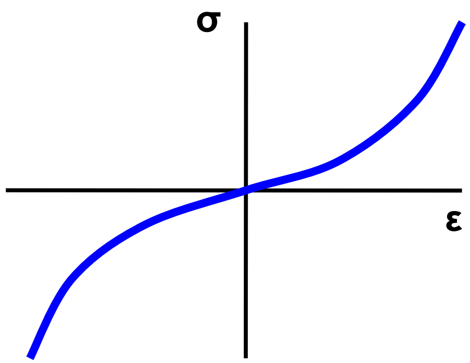


  - Not the same as viscous drag!

Non-Linearity



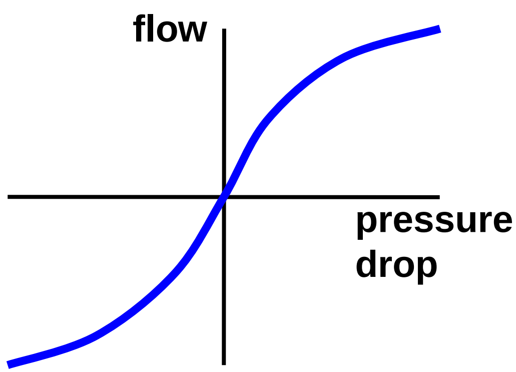
- Material non-linearity
  - Stress is not proportional to strain
  - Load is not proportional to extension

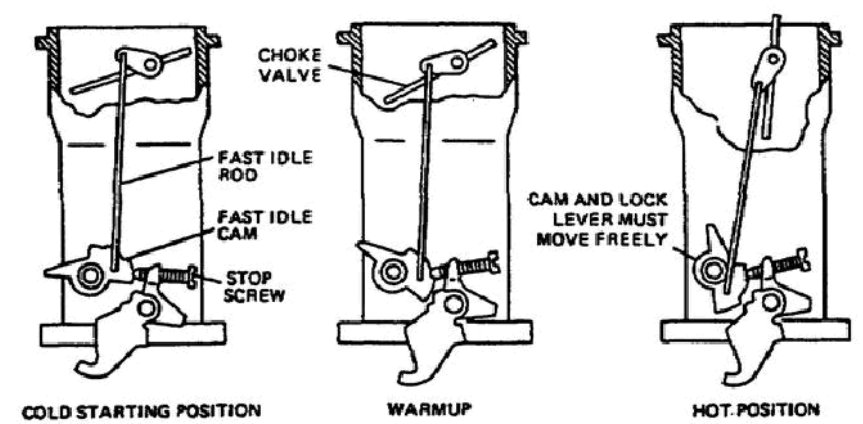


  - Natural and synthetic rubbers, elastic materials, etc.

Non-Linearity

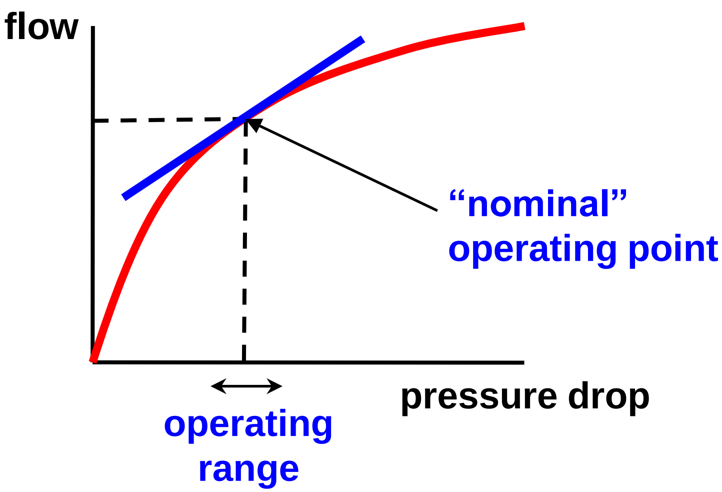
- Flow through an orifice
  - Also known as choked flow
  - Used for flow measurement or controlling the mixture in an IC engine





Linearisation

- We normally try to keep systems close to an operating point
  - Most efficient (particularly in steam or gas turbines)



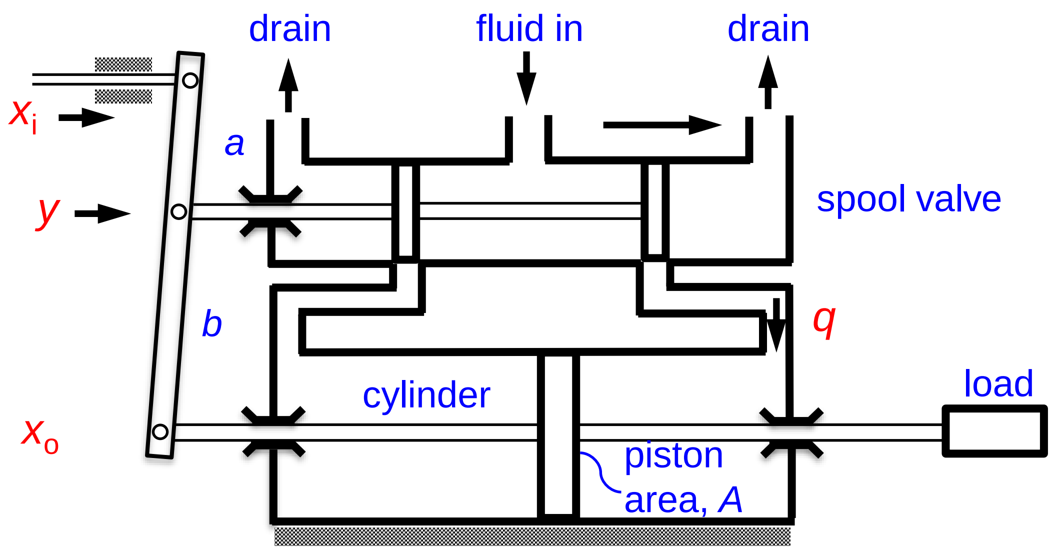
System behaviour approximates to a linear relationship near the "nominal" operating point:

$$y=mx+b$$

Brilliant idea no. 2

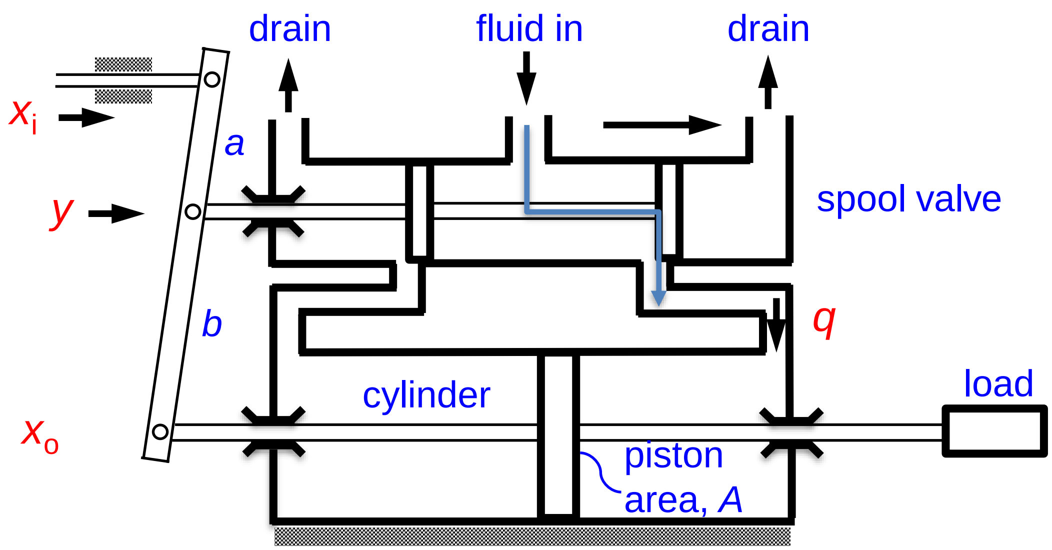
- Hydraulic Position Control
  - Also known as servo-assistance
  - Aeroplane flaps
  - Car brakes
  - Power steering (some cars)
  - Tractors and JCBs!

Hydraulic Position Control System



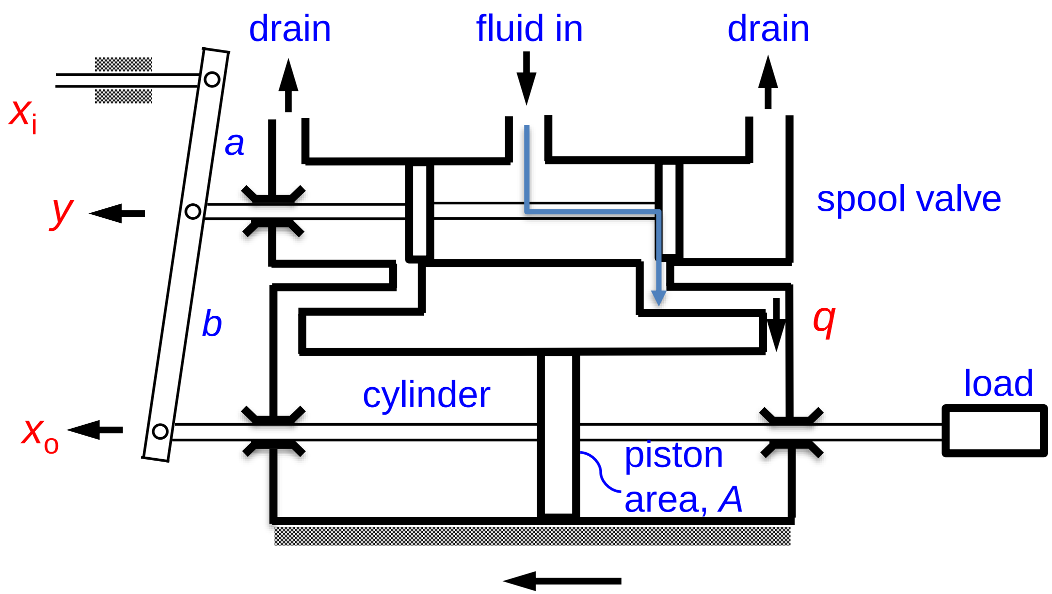
- How it works
  - Operator changes setting (x_i)
  - Piston is fulcrum – spool valve (y) translates
  - Spool valve admits fluid into cylinder

Hydraulic Position Control System



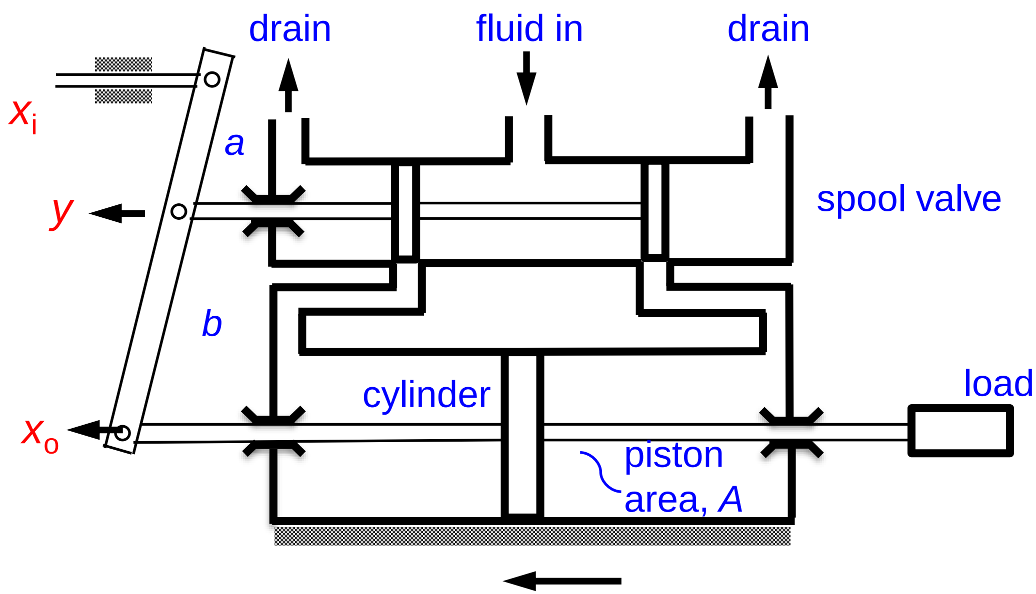
- How it works
  - Operator changes setting (x_i)
  - Piston is fulcrum – spool valve (y) translates
  - Spool valve admits fluid into cylinder

Hydraulic Position Control System



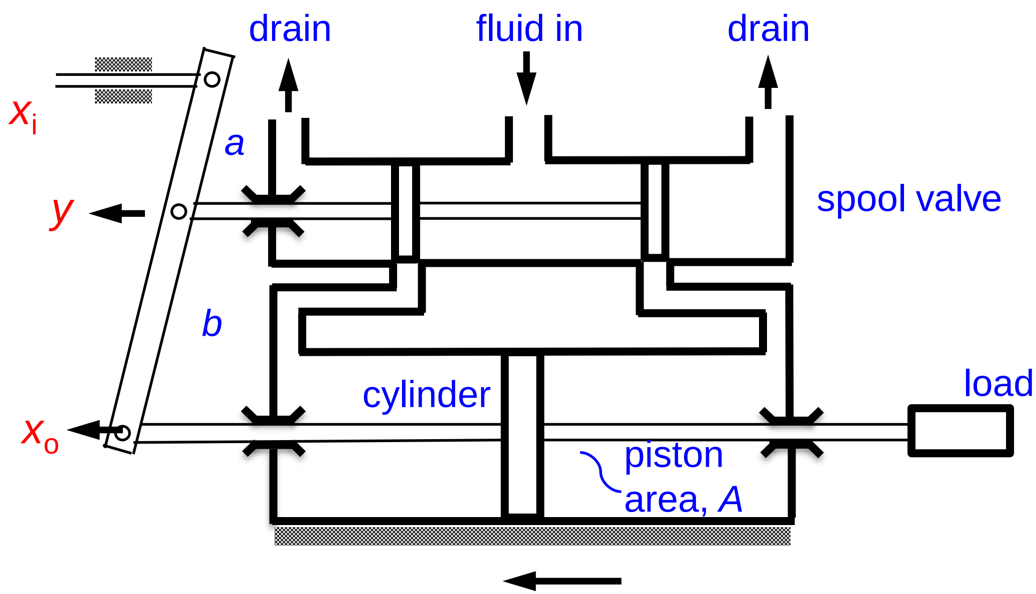
- How it works (continued)
  - Spool valve admits fluid into cylinder
  - Input (xi) is fulcrum - Piston moves until valve closes

Hydraulic Position Control System



- How it works (continued)
  - Spool valve admits fluid into cylinder
  - Input (xi) is fulcrum - Piston moves until valve closes

Hydraulic Position Control System

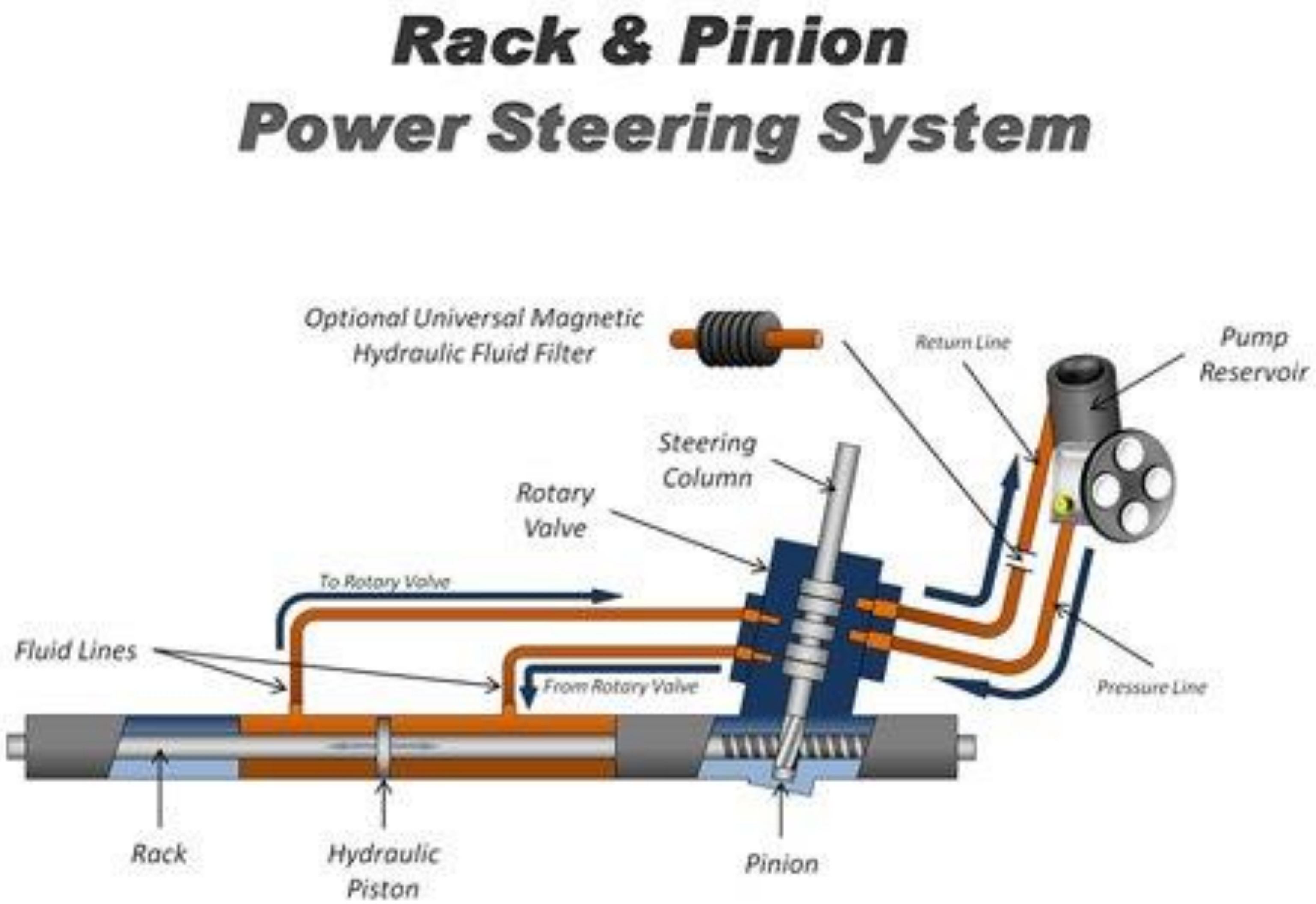


- Why is it brilliant?
  - Enormous amplification of force
  - It is possible (but very difficult) to move manually in case of power failure

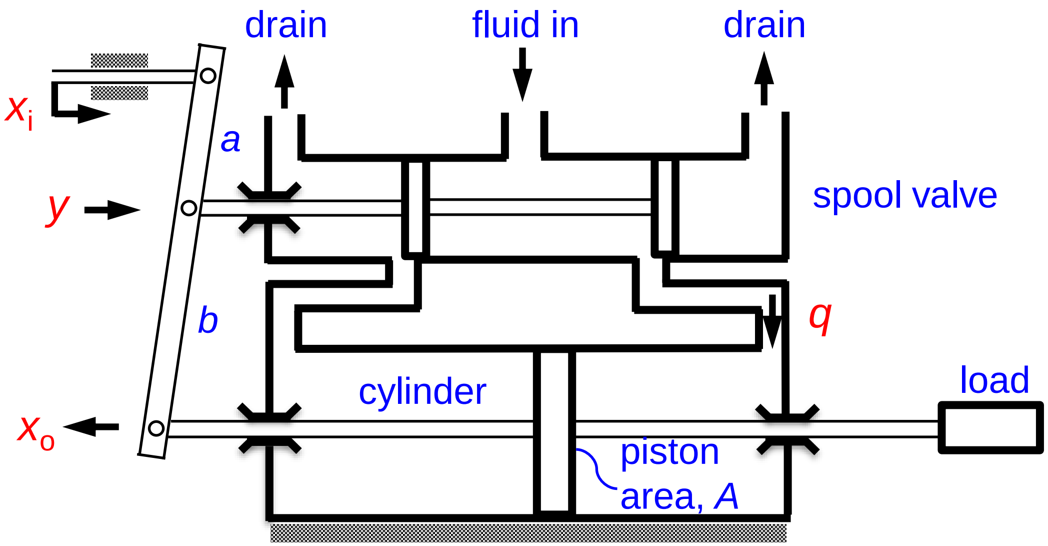
Hydraulic Position Control

- The system here describes linear displacement … how would you use a hydraulic system for angular displacement?

Hint


Rack & Pinion
Power Steering System
Optional Universal Magnetic
Hydraulic Fluid Filter
Return Line
Pump
Reservoir
Steering
Column
Rotary
Valve
To Rotary Valve
Fluid Lines
From Rotary Valve
Pressure Line
Rack
Hydraulic
Piston
Pinion

Case Study: Hydraulic Position Control System

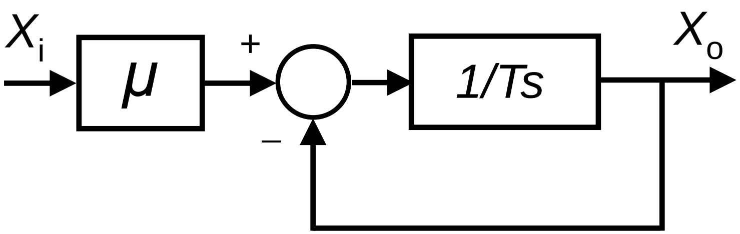


Show that the **transfer function** may be written as

$$G(s) = \frac{X_o(s)}{X_i(s)} = \frac{\mu}{1 + Ts}$$

1st order system

with the **block diagram**

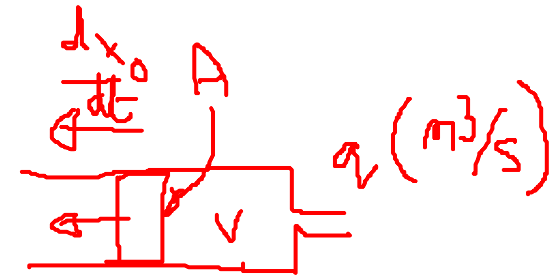


Hydraulic Position Control System: Equations for the Model

Spool Valve

in the time domain $$q = Ky$$

transfer function $$\frac{Q(s)}{Y(s)} = K \qquad (1)$$



Ram Piston

in the time domain $$A\frac{dx_o}{dt} = q$$

transfer function $$\frac{X_o(s)}{Q(s)} = \frac{1}{As} \qquad (2)$$

$$q = \frac{dV}{dt} = A\frac{dx_o}{dt}$$

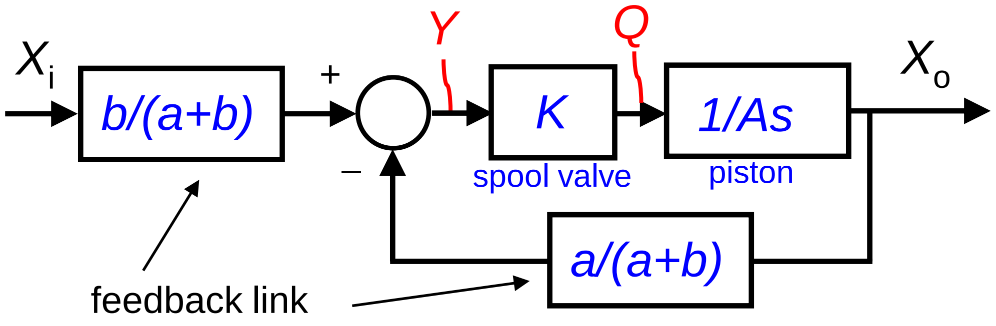
$$Q(s) = A s X_o(s)$$

Feedback Link

in the time domain $$y = \frac{b}{a+b}x_i - \frac{a}{a+b}x_o$$

transfer function $$Y(s) = \frac{b}{a+b}X_i(s) - \frac{a}{a+b}X_o(s) \qquad (3)$$

Hydraulic Position Control System: Overall Transfer Function



From the block diagram

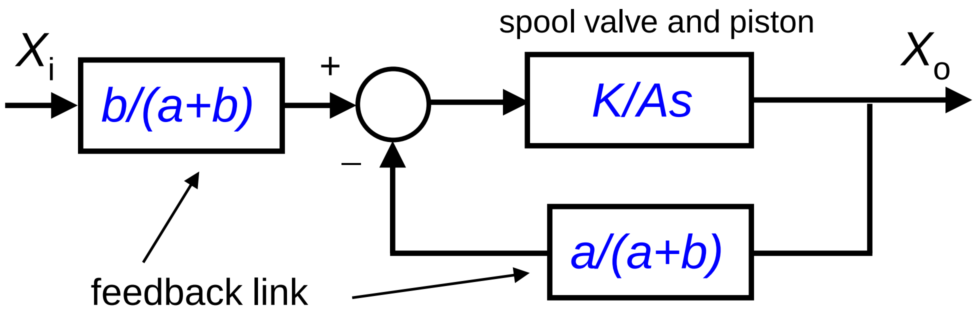
$$X_o(s) = \left[X_i(s)\frac{b}{a+b} - X_o(s)\frac{a}{a+b}\right]\frac{K}{As}$$

rearranging

$$\left[1 + \frac{A(a+b)s}{Ka}\right]X_o(s) = \frac{b}{a}X_i(s)$$

$$\frac{X_0(s)}{X_i(s)} = \frac{\frac{b}{a}}{1 + \left(\frac{A(a+b)}{Ka}\right)s}$$

Hydraulic Position Control System: Overall Transfer Function



From the block diagram

$$X_o(s) = \left[X_i(s)\frac{b}{a+b} - X_o(s)\frac{a}{a+b}\right]\frac{K}{As}$$

rearranging

$$\left[1 + \frac{A(a+b)s}{Ka}\right]X_o(s) = \frac{b}{a}X_i(s)$$

$$\frac{X_0(s)}{X_i(s)} = \frac{\frac{b}{a}}{1 + \left(\frac{A(a+b)}{Ka}\right)s}$$

Hydraulic Position Control System: Control System Model

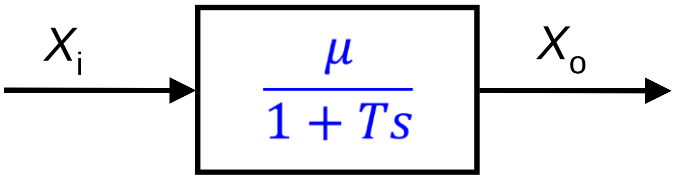
$$\frac{X_0(s)}{X_i(s)} = \frac{\frac{b}{a}}{1 + \left(\frac{A(a+b)}{Ka}\right)s}$$

This can be rewritten as a First order system with time constant *T* and gain *μ*

$$\frac{X_\mathrm{o}(s)}{X_\mathrm{i}(s)} = \frac{\mu}{1 + Ts} \qquad (4)$$

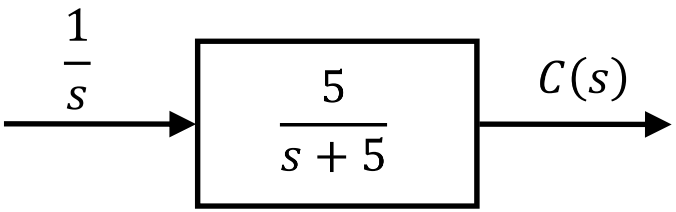
$$T = \frac{A(a+b)}{Ka}$$ **time constant**

$$\mu = \frac{b}{a}$$ **steady-state gain**



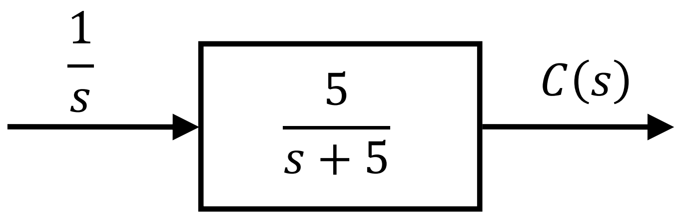
Example

- A first order system is shown below:



- Find the output response $c(t)$, time constant, rise time and settling time for the system

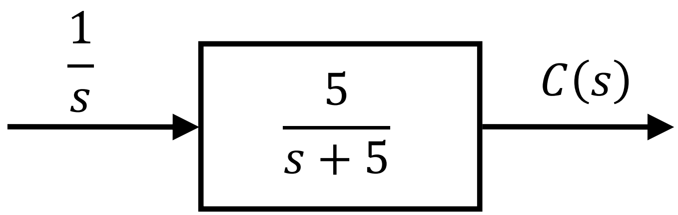
Example



- $C(s) = \frac{5}{s(s+5)} = \frac{5}{s}\left(\frac{1}{1+0.2s}\right)$
- Time constant = 0.2
- From table of Laplace transforms:

		s^2
7	e^{-at}	$\frac{1}{s+a}$
8	$1 - e^{-at}$	$\frac{a}{s(s+a)}$

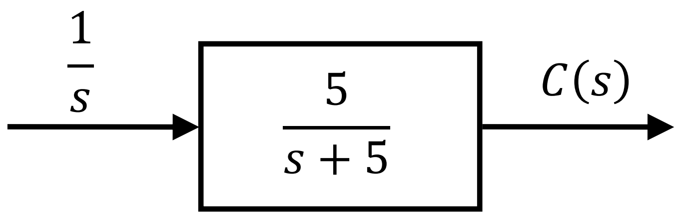
Example



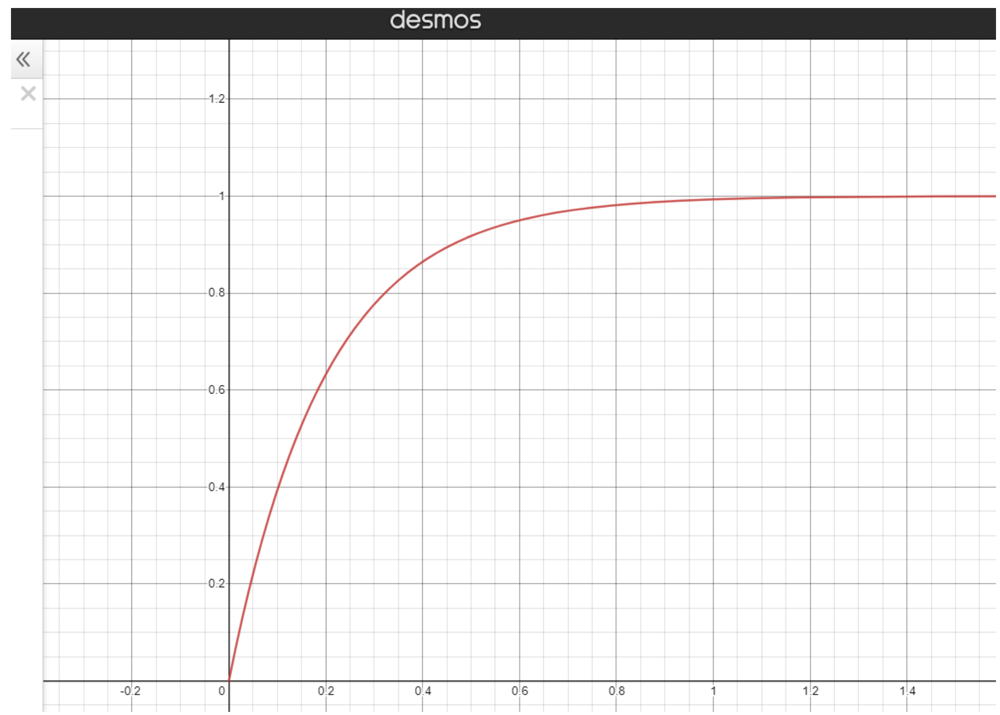
- $C(s) = \frac{5}{s(s+5)} = \frac{5}{s}\left(\frac{1}{1+0.2s}\right)$
- $c(t) = 1 - e^{-5t} = 1 - e^{-t/0.2}$

		s^2
7	e^{-at}	$\frac{1}{s+a}$
8	$1 - e^{-at}$	$\frac{a}{s(s+a)}$

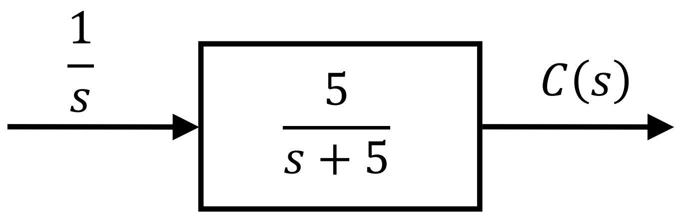
Example



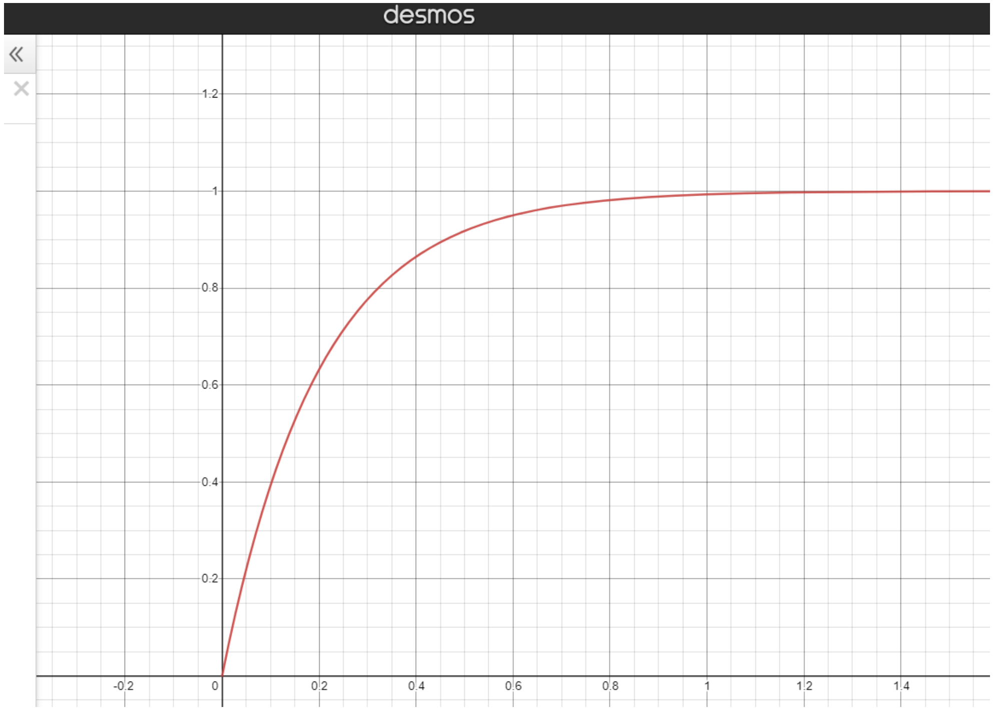
- $C(s) = \dfrac{5}{s(s+5)} = 5\left(\dfrac{1}{1+0.2s}\right)$
- $c(t) = 1 - e^{-5t} = 1 - e^{-t/0.2}$



Example



- Rise time?
- Settling time?

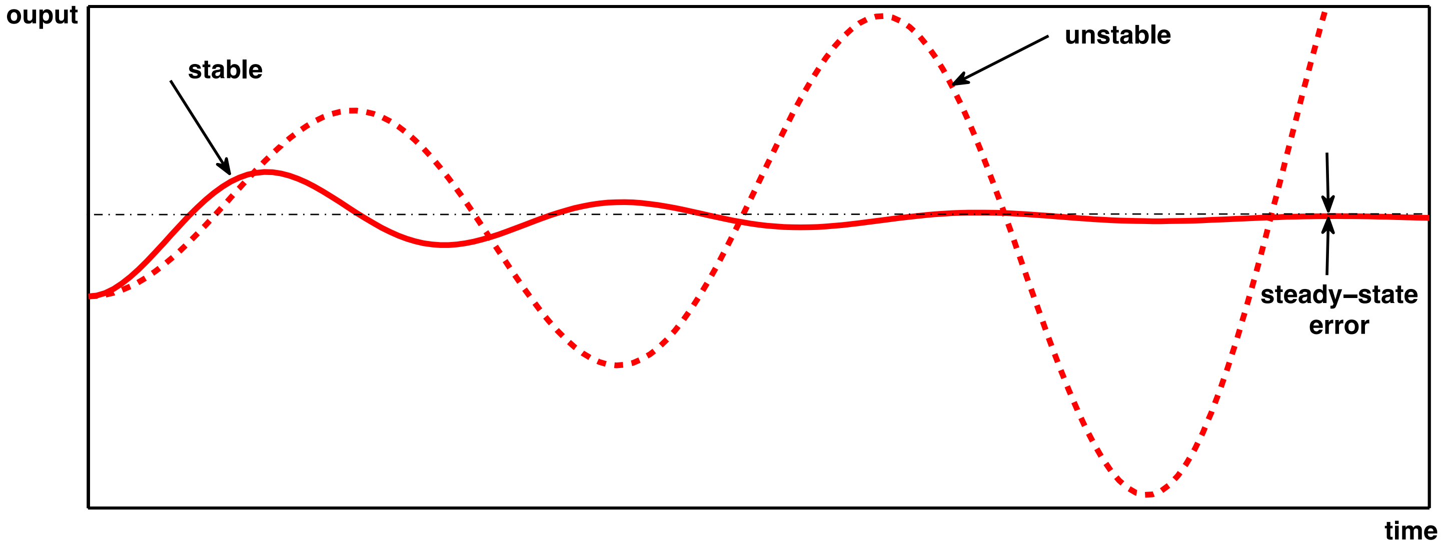


Stability

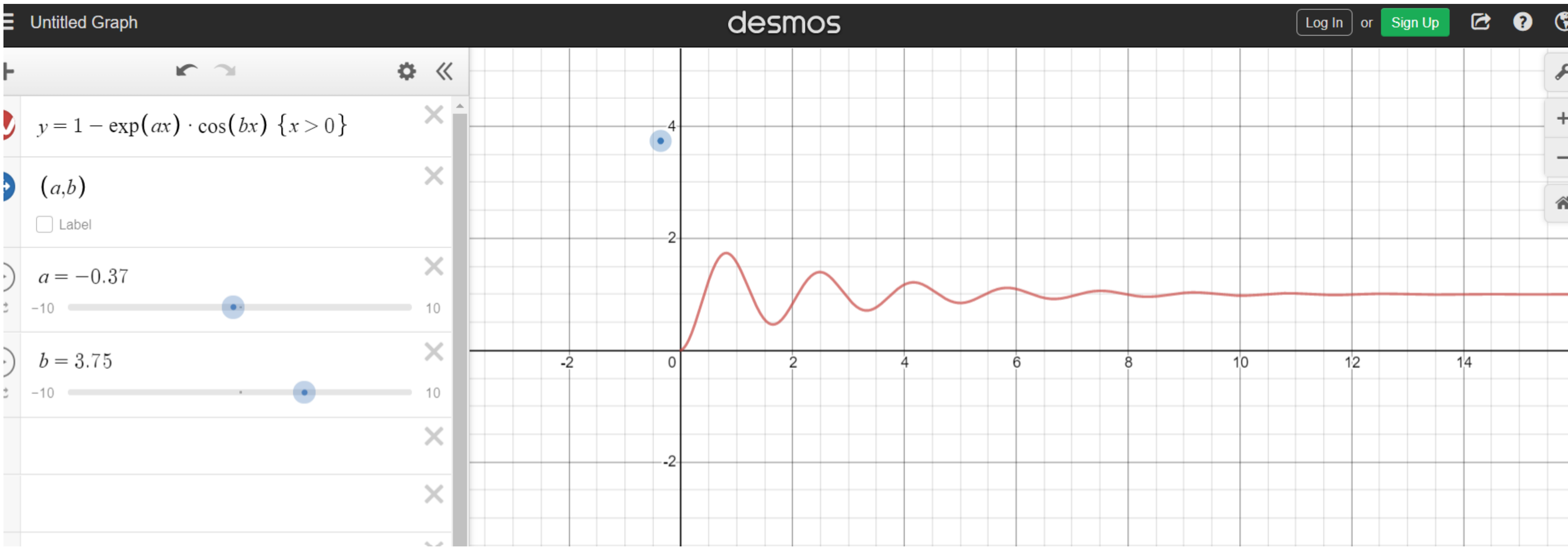
Why is stability important?

- Active systems have their own power input
  - https://www.youtube.com/watch?v=-LFLV47VAbI
- Passive systems need to be excited at their resonant frequency
  - https://www.youtube.com/watch?v=nFzu6CNtqec
- Designers need to know how the system will behave:

i) **Is the System Stable?**



ii) **How Accurate is the System in Steady State?**


Untitled Graph
desmos
Log In
or
Sign Up
y = 1 − exp(ax) · cos(bx) {x > 0}
(a,b)
Label
a = −0.37
−10
10
b = 3.75
−10
10

Introduction to Transient and Steady-State Responses

Subject control systems to **standard inputs**, compare and tune their performance.

We will consider three such inputs:

i) step input

ii) ramp input (linear change with time)

iii) harmonic input (considered in Vibration).

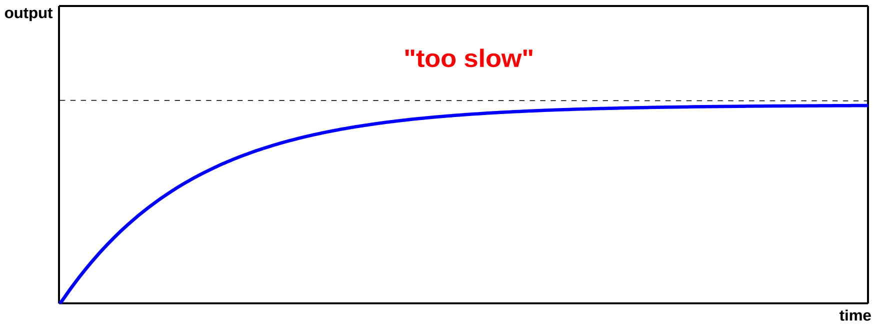
These inputs are useful because:

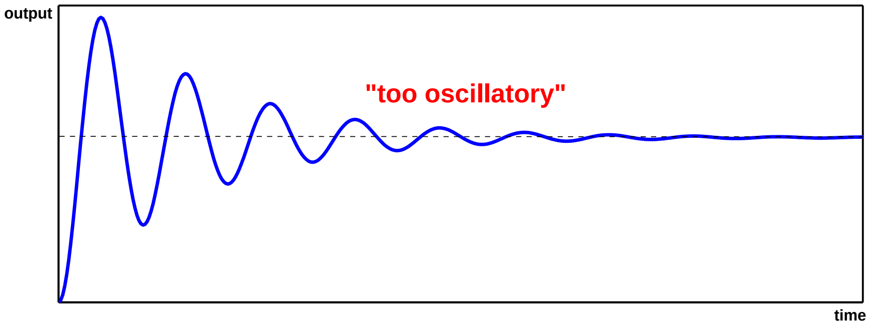
a) easy to apply in practice, both theoretically and experimentally;

b) approximate to operating conditions in control systems.

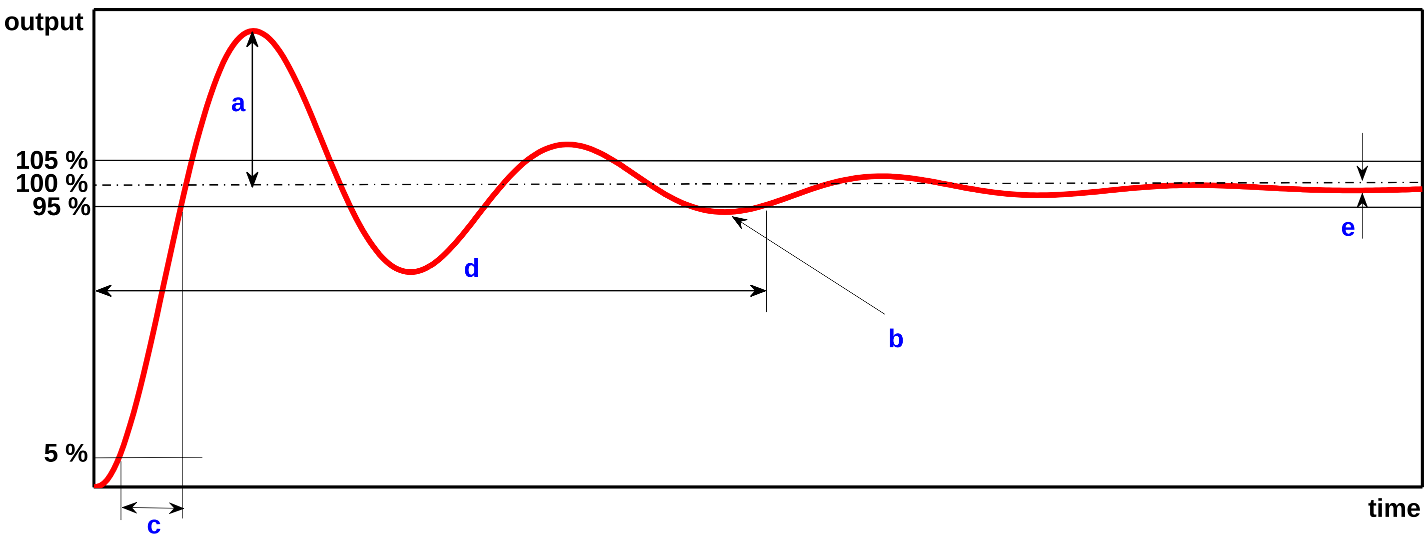
Other forms possible (e.g. Impulsive and Random), not considered in module.

iii) **How Quickly Does the System Reach a Steady State?**





Practical Measures of Transient Response



a) Maximum Overshoot as a percentage of step size.

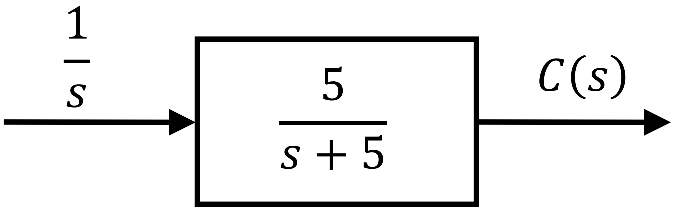
b) Number of Oscillations before system settles to within a fixed percentage (5% say) of its steady state value.

c) Rise Time: The time taken for output to rise from 5% to 95% of step size.

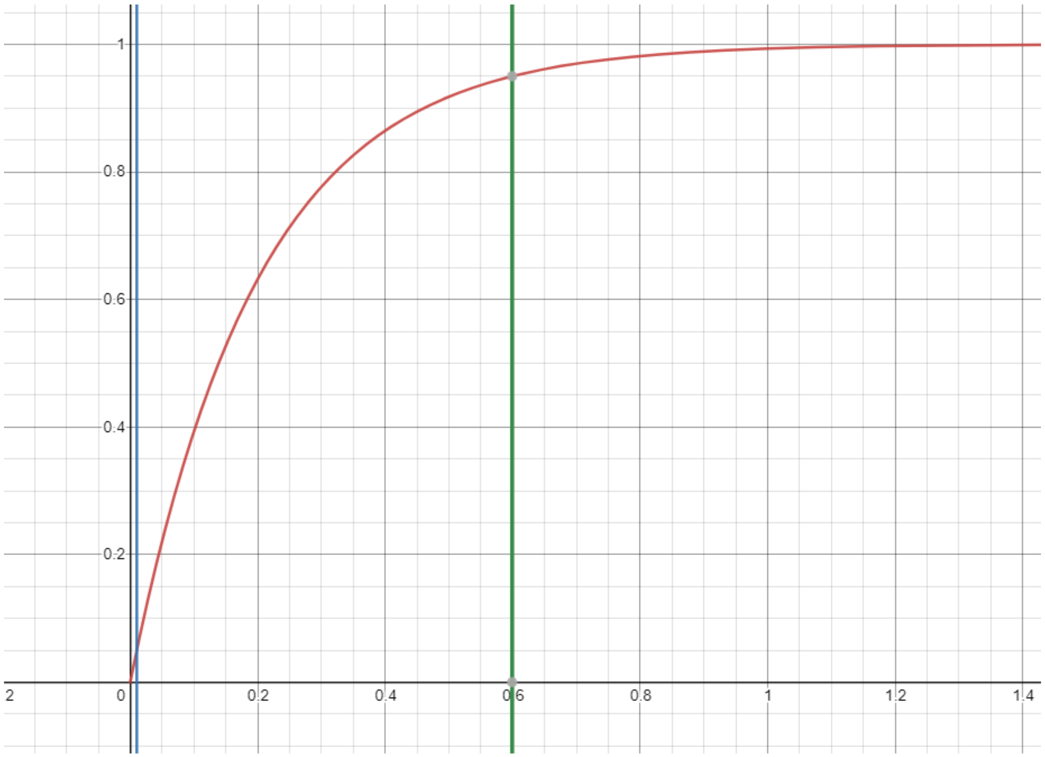
d) Settling Time: The time taken for output to reach and remain within ±5% of steady state value.

e) Steady State Error

Example



- Rise time?
  - From c=0.5 to 0.95
  - 0.01s to 0.6s or 0.59s
- Settling time?
  - To 0.95 in this case
  - 0.60s



Seminar preparation

- Recap of this material
- Example sheet 3 Question 2
- Exam question from 2019 (no. 4)

What Next?

- Block Diagram Representation and Manipulation
- Hydraulic Position Control System (continued): first order system
- Electro-Mechanical Position Control System: second order system

Finis

- Any questions so far?