Asymmetrical Bending

Worked Example 1 – Principal Axes & Principal 2nd Moments of Area

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

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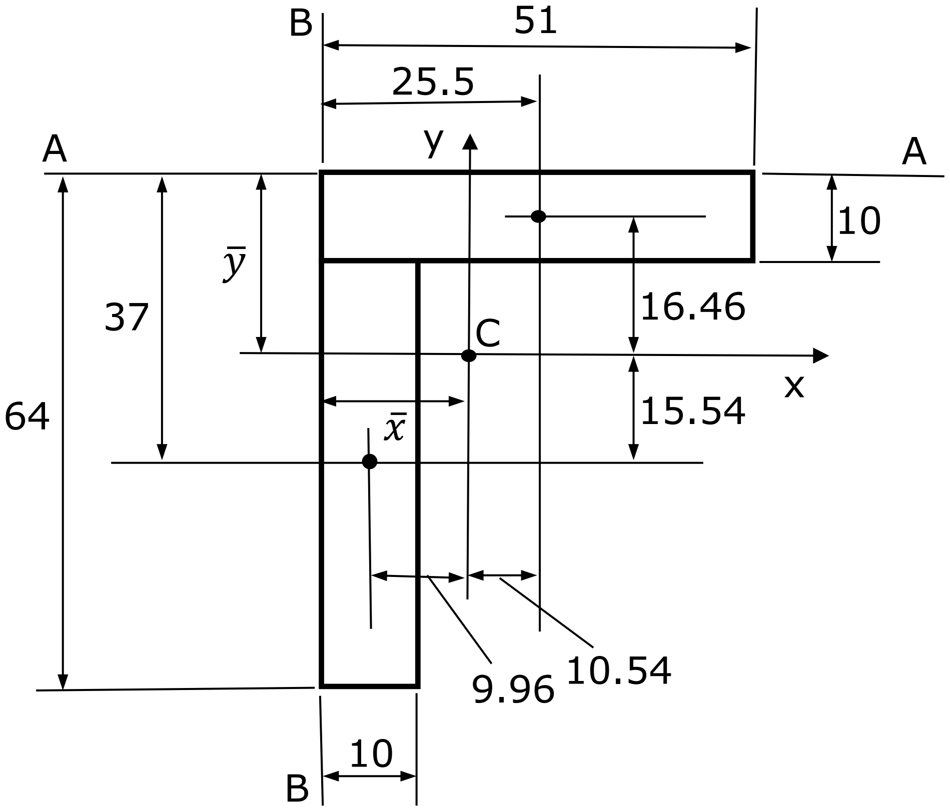
Worked Example 1

Principal Axes & Principal 2nd Moments of Area

For the beam cross-section shown below, calculate:

- The Principal 2nd Moments of Area
- The directions of the Principal Axes



All dimensions in mm

Solution

Total Area: $A = (51{\times}10) + (10{\times}54) = 1050\ \text{mm}^2$

Position of Centroid

Moments about AA:

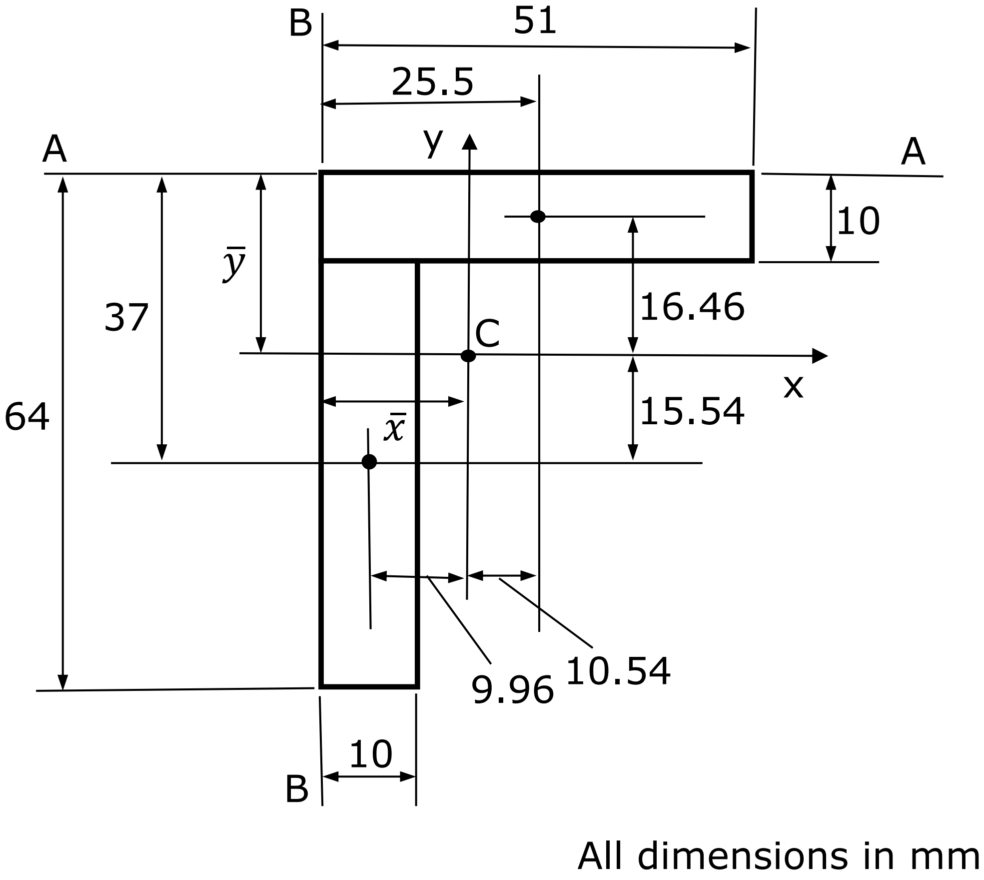
$$1050{\times}\bar{y} = \big((51{\times}10){\times}5\big) + \big((10{\times}54){\times}37\big)$$

$$\therefore\ \bar{y} = 21.46\ \text{mm}$$

Moments about AA:

$$1050{\times}\bar{x} = \big((10{\times}51){\times}25.5\big) + \big((54{\times}10){\times}5\big)$$

$$\therefore\ \bar{x} = 14.96\ \text{mm}$$



2nd Moments of Area

Parallel Axis Theorem:

$$I_{x'} = (I_x + Ab^2)_a + (I_x + Ab^2)_b$$

$$= \left(\frac{51\times10^3}{12} + 51\times10\times16.46^2\right) + \left(\frac{10\times54^3}{12} + 10\times54\times-15.54^2\right)$$

$$= 404{,}051\ \text{mm}^4$$

$$I_{y'} = \left(I_y + Aa^2\right)_a + \left(I_y + Aa^2\right)_b$$

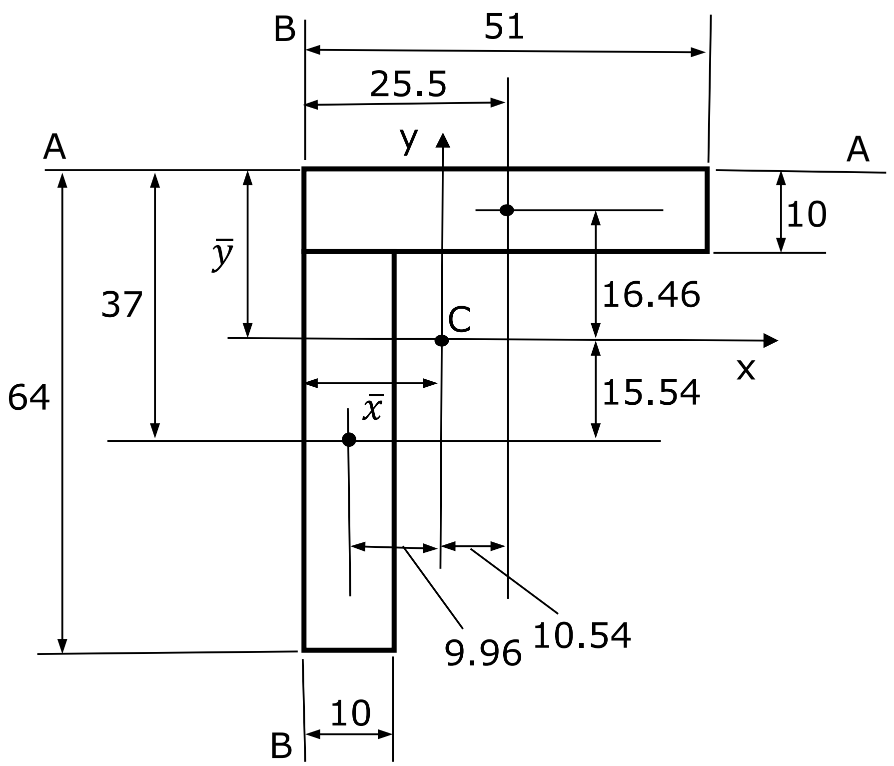
$$= \left(\frac{10\times51^3}{12} + 10\times51\times10.54^2\right) + \left(\frac{54\times10^3}{12} + 54\times10\times-9.96^2\right)$$

$$= 225{,}268\ \text{mm}^4$$

$$I_{x'y'} = \left(I_{xy} + abA\right)_a + \left(I_{xy} + abA\right)_b$$

$$= (0 + 51\times10\times10.54\times16.64) + (0 + 54\times10\times-9.96\times-15.54)$$

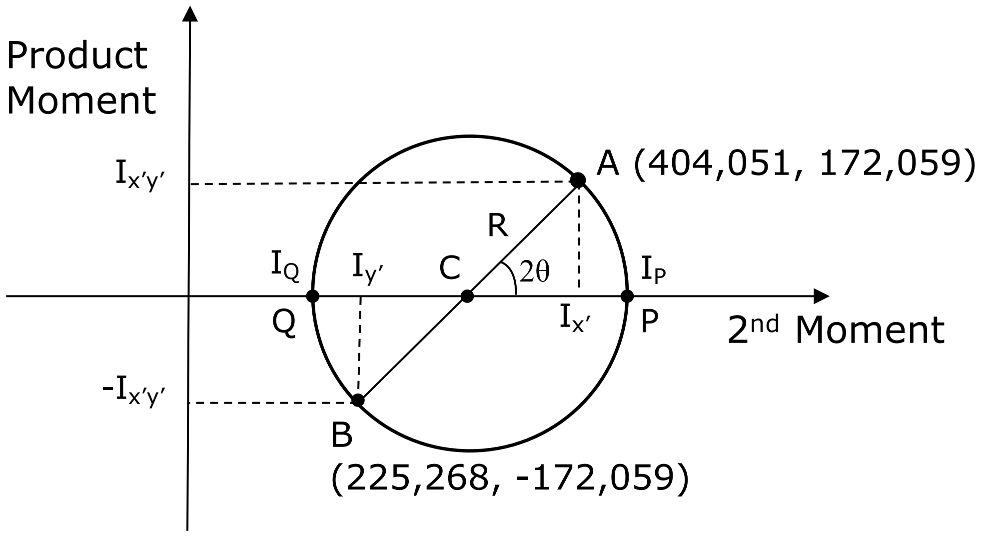
$$= 172{,}059\ \text{mm}^4$$



All dimensions in mm

$I_{x'} = 404{,}051 \text{ mm}^4$ $\quad$ $I_{y'} = 225{,}268 \text{ mm}^2$ $\quad$ $I_{x'y'} = 172{,}059 \text{ mm}^4$

Mohr's Circle



$$C = \frac{I_{x'} + I_{y'}}{2} = 314{,}659 \text{ mm}^4$$

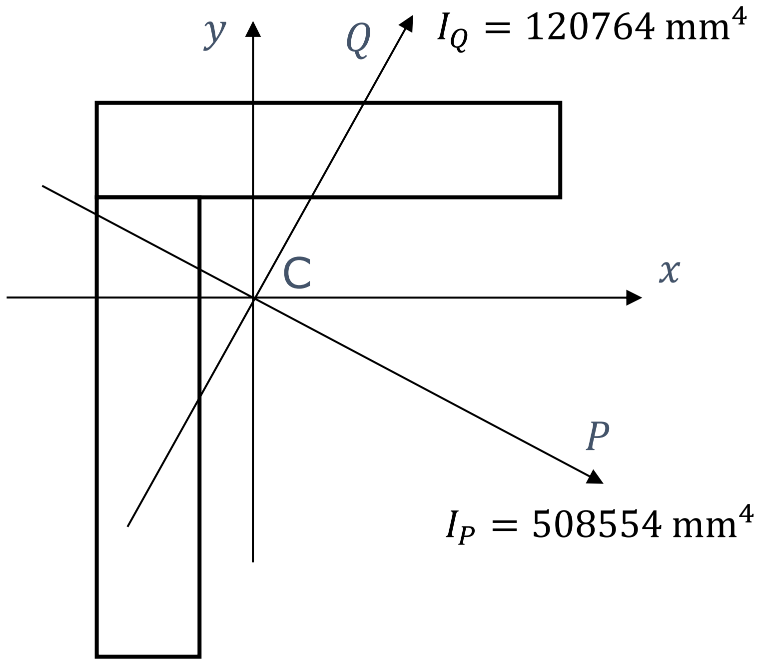
$$R = \sqrt{\left(\frac{I_{x'} - I_{y'}}{2}\right)^2 + {I_{x'y'}}^2} = 193{,}895 \text{ mm}^4$$

$$I_p = C + R = 508{,}554 \text{ mm}^4$$

$$I_q = C - R = 120{,}764 \text{ mm}^4$$

$$\sin 2\theta = \frac{I_{x'y'}}{R} = \frac{172{,}059}{193{,}895} = 0.887$$

$$\theta = 31.27°$$



If the beam was loaded in the direction of the P or Q axis

- there would be no Product Moment of Area
- the beam would deflect only in this direction