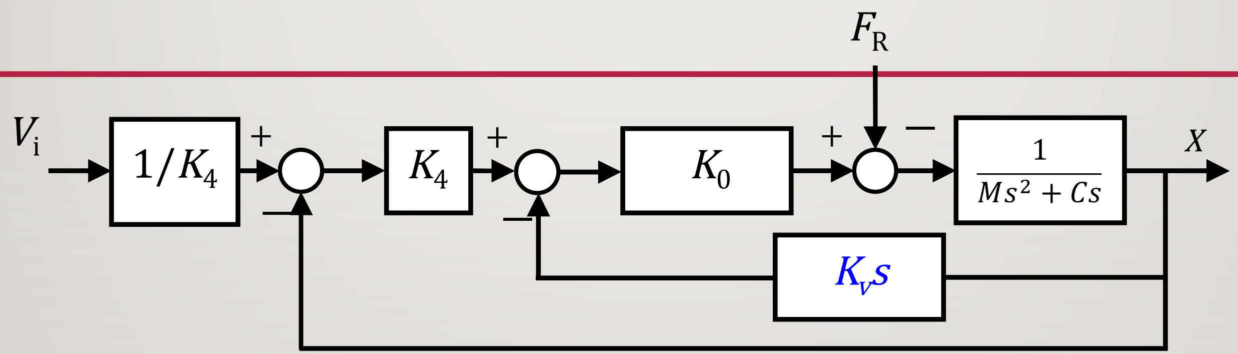
DYNAMICS AND CONTROL

CONTROL CONSOLIDATION 3

GENERAL INTRODUCTION – SESSION 3

- Second order systems and PID controllers – additional material
- Routh Hurwitz – worked examples
- Q&A

a) Velocity Feedback



The governing equation follows as:

$$[Ms^2 + (C + K_0 K_v)s + K_0 K_4]X(s) = K_0 K_4 X_i(s) - F_R(s)$$

$$s^2 + 2\gamma\omega_n s + \omega_n^2 = 0$$

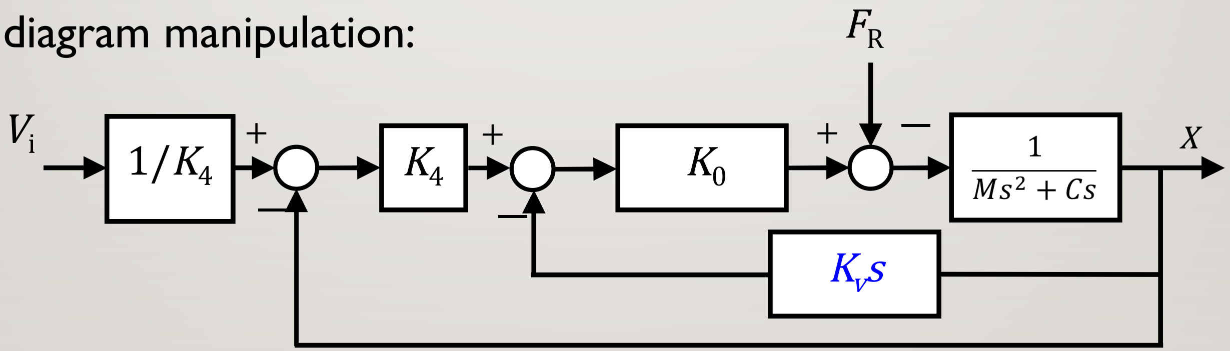
velocity feedback ➔ viscous damping

The Velocity lag ($F_R = 0$) under ramp remains

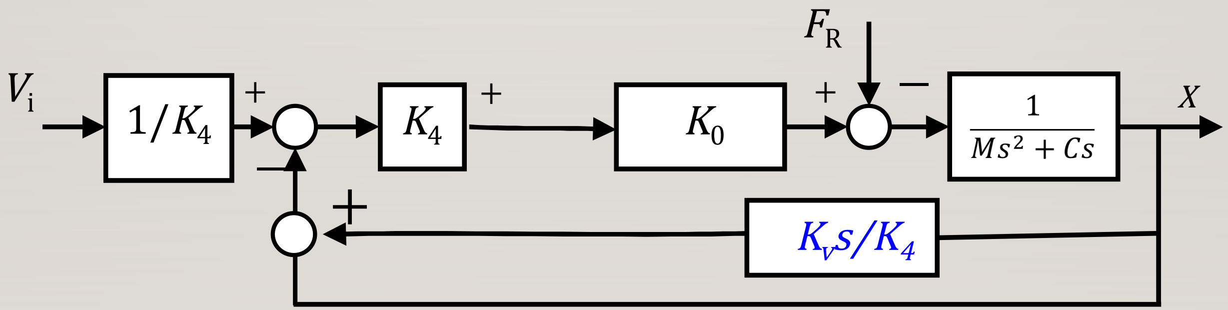
$$e_{ss} = \frac{[C + K_0 K_v]}{K_0 K_4}\Omega_x$$

VELOCITY FEEDBACK

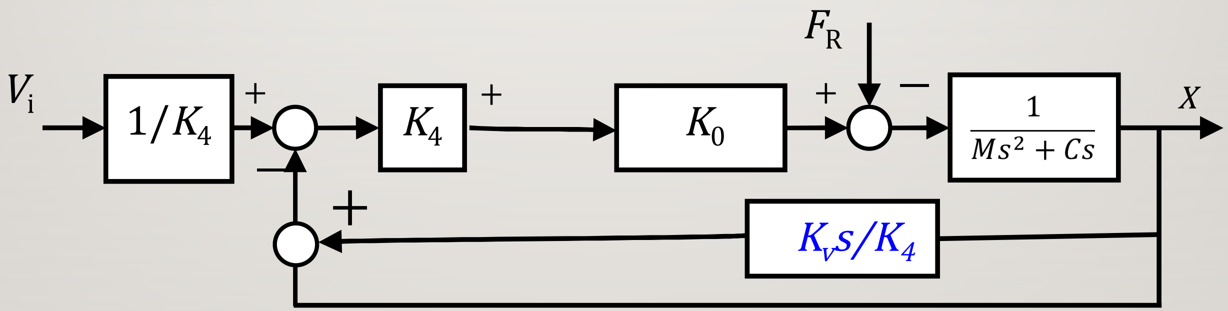
- Use block diagram manipulation:



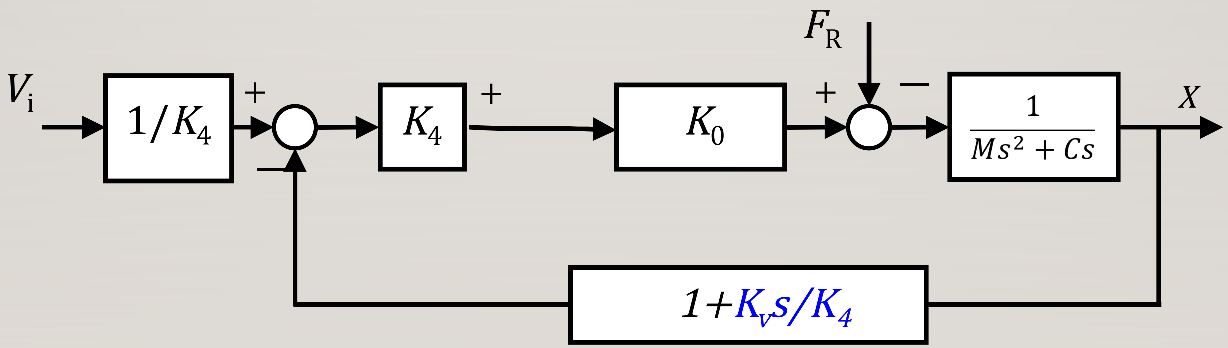
- Becomes:



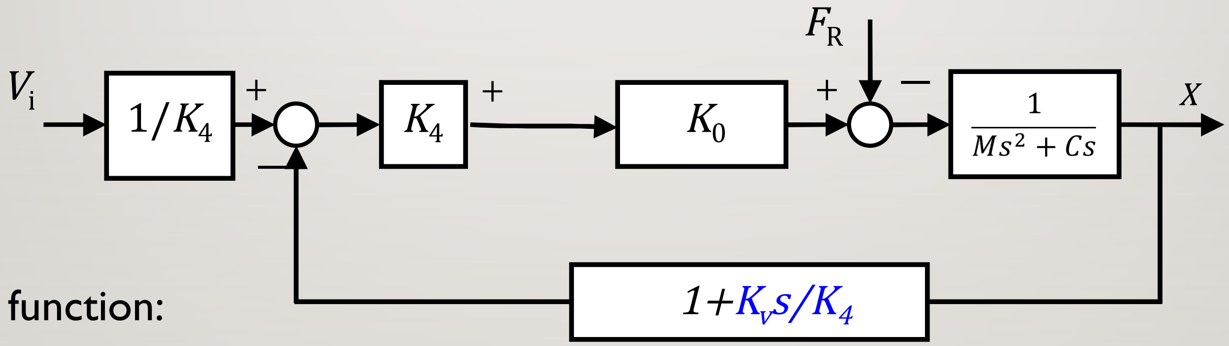
VELOCITY FEEDBACK



- Becomes:

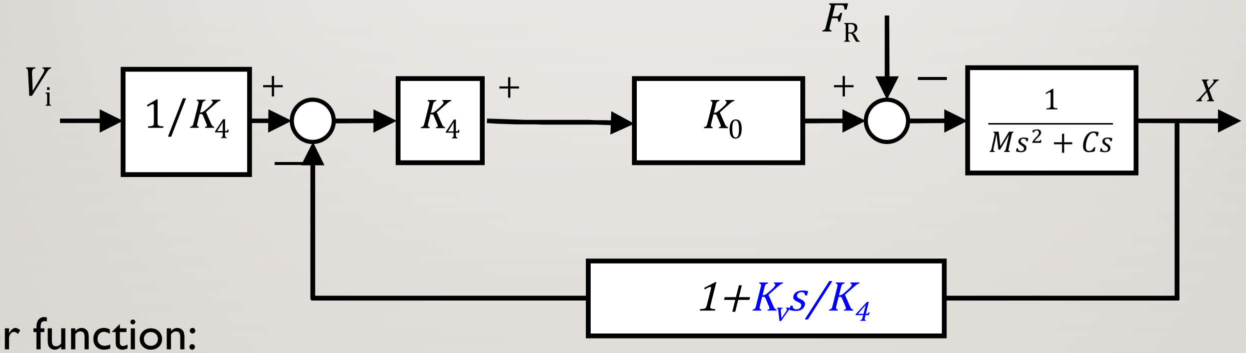


VELOCITY FEEDBACK



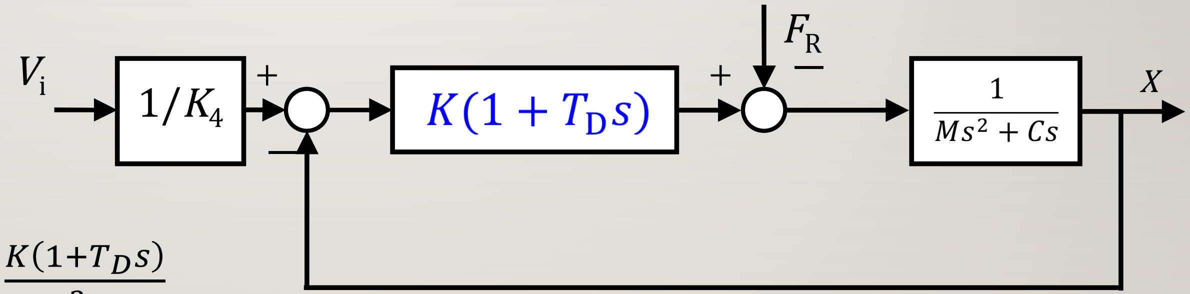
- Transfer function:
- $\frac{V_i(s)}{K_4} = X_i(s)$
- $X(s) = \left(X_i(s) - \left(1 + \frac{K_v s}{K_4}\right) X(s)\right)\left(\frac{K_4 K_0}{Ms^2+Cs}\right)$

VELOCITY FEEDBACK



- Transfer function:
- $X(s)(Ms^2 + Cs) + (K_4K_0 + K_0K_Vs)X(s) = K_4K_0X_i(s)$
- $X(s)(Ms^2 + (C + K_0K_V)s + K_4K_0) = K_4K_0X_i(s)$
- $\frac{X(s)}{X_i(s)} = \frac{K_4K_0}{Ms^2+(C+K_VK_0)s+K_4K_0} = \frac{\omega^2}{(s^2+2\gamma\omega s+\omega^2)}$ $\qquad \omega^2 = \frac{K_4K_0}{M}$

Proportional and Derivative Control (P+D)



- Transfer function:
- $X(s) = \left(X_i(s) - X(s)\right)\frac{K(1+T_D s)}{Ms^2+Cs}$
- $X(s)\left(Ms^2 + Cs + K(1 + T_D s)\right) = X_i(s)K(1 + T_D s)$

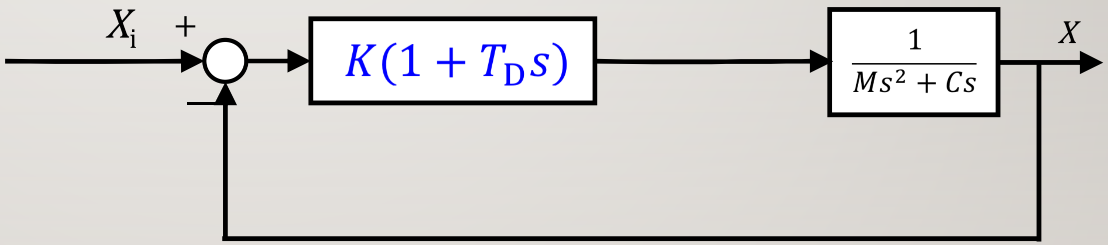
Notes:

i) Damping increased without increasing power consumption (why?)

ii) Overshoot is decreased

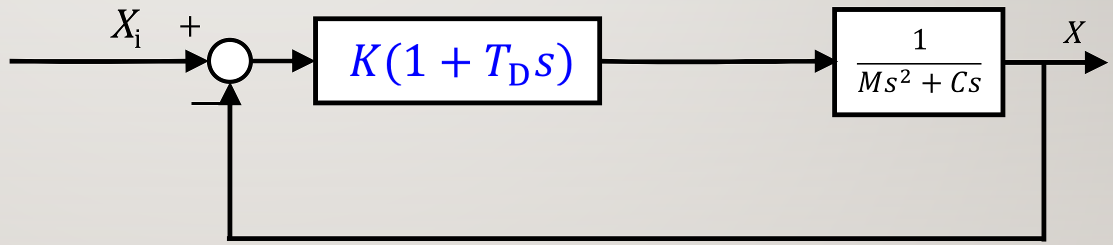
iii) Derivative control has no effect on steady state error.

Proportional and Derivative Control (P+D)
Steady state error for ramp input $\frac{\Omega}{s^2}$



- $X(s) = \frac{\Omega}{s^2}\frac{(1+T_D s)}{(Ms^2+(C+KT_D)s+K)}$ $X_i(s) = \frac{\Omega}{s^2}$ $E(s) = X_i(s) - X(s)$
- Final Value Theorem: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$
- Take 5 – have a go …

Proportional and Derivative Control (P+D)
Steady state error for ramp input $\frac{\Omega}{s^2}$

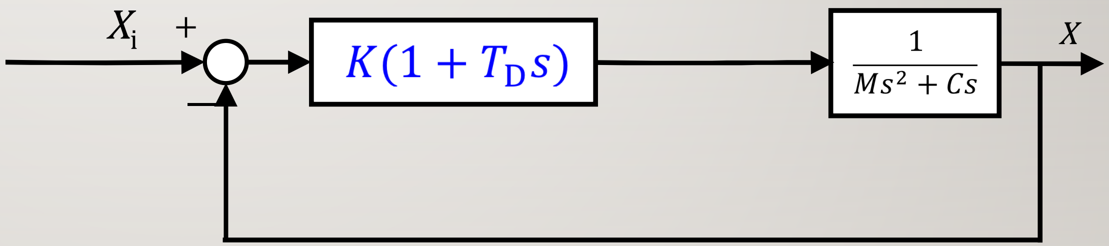


- $X(s) = \frac{\Omega}{s^2}\frac{(1+T_D s)}{(Ms^2+(C+KT_D)s+K)}$ $\qquad E(s) = \frac{\Omega}{s^2} - \frac{\Omega}{s^2}\frac{K(1+T_D s)}{(Ms^2+(C+KT_D)s+K)}$
- Final Value Theorem: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{s\Omega\left(Ms^2+(C+KT_D)s+K\right)}{s^2(Ms^2+(C+KT_D)s+K)} - \frac{s\Omega}{s^2}\frac{K(1+T_D s)}{(Ms^2+(C+KT_D)s+K)}$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{s\Omega}{s^2}\frac{\left(Ms^2+(C+KT_D)s+K\right)-K(1+T_D s)s}{(Ms^2+(C+KT_D)s+K)} = \frac{s\Omega\left(Ms^2+Cs\right)}{s^2(Ms^2+(C+T_D)s+K)}$$

Proportional and Derivative Control (P+D)

Steady state error for ramp input $\frac{\Omega}{s^2}$

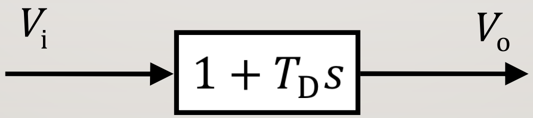


$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{s\Omega}{s^2}\frac{\left(Ms^2+(C+KT_D)s+K\right)-K(1+T_D s)s}{(Ms^2+(C+KT_D)s+K)} = \frac{s\Omega\left(Ms^2+Cs\right)}{s^2(Ms^2+(C+T_D)s+K)}$$

- Apply the limit: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{s\Omega\left(Ms^2+Cs\right)}{s^2(Ms^2+(C+KT_D)s+K)} = \frac{C}{K}$
- Therefore differential control cannot completely remove the S-S error

Proportional and Derivative Control (P+D)

iv) Derivative action tends to amplify ‘noise’ in the system:



in time domain $$v_\mathrm{o}(t) = v_\mathrm{i}(t) + T_\mathrm{D}\frac{\mathrm{d}v_\mathrm{i}(t)}{\mathrm{d}t}$$

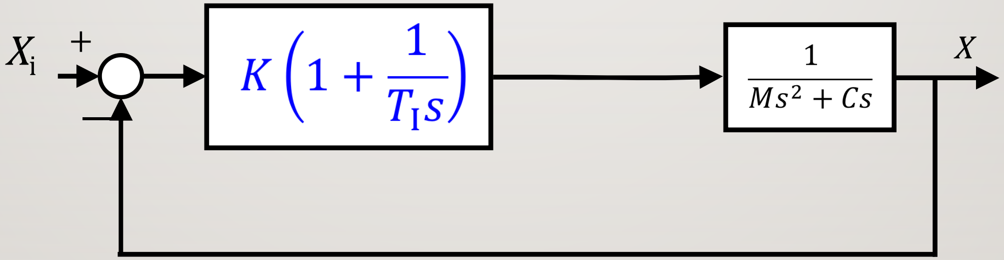
If the input signal is $$v_\mathrm{i}(t) = V + v_\mathrm{n}\sin(\omega t)$$

the output is $$v_\mathrm{o}(t) = V + v_\mathrm{n}\sin(\omega t) + T_\mathrm{D}\omega v_\mathrm{n}\cos(\omega t)$$

Now, if $\omega \gg 1/T_\mathrm{D}$ noise is amplified $T_\mathrm{D}\omega v_\mathrm{n} \gg v_\mathrm{n}$

Proportional and Integral Control (P+I)

Proportional error is modified by adding an integral of error.
This can also be carried out electronically.

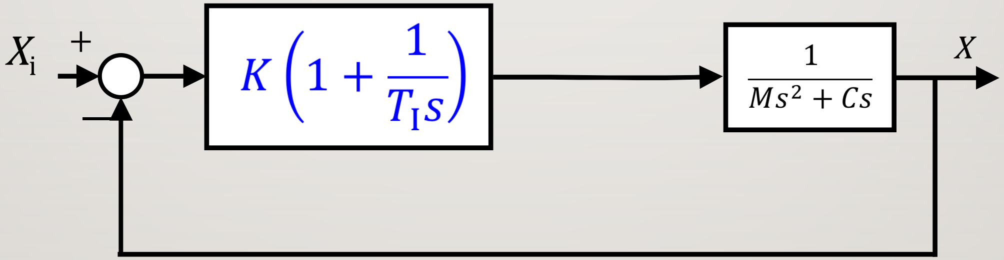


Governing equation:

$$X(s) = \left(X_i(s) - X(s)\right)\left(Ks + \frac{K}{T_i}\right)\left(\frac{1}{Ms^3 + Cs^2}\right)$$

$$X(s)\left(1 + \left(Ks + \frac{K}{T_i}\right)\left(\frac{1}{Ms^3 + Cs^2}\right)\right) = X_i(s)\left(Ks + \frac{K}{T_i}\right)\left(\frac{1}{Ms^3 + Cs^2}\right)$$

Proportional and Integral Control (P+I)

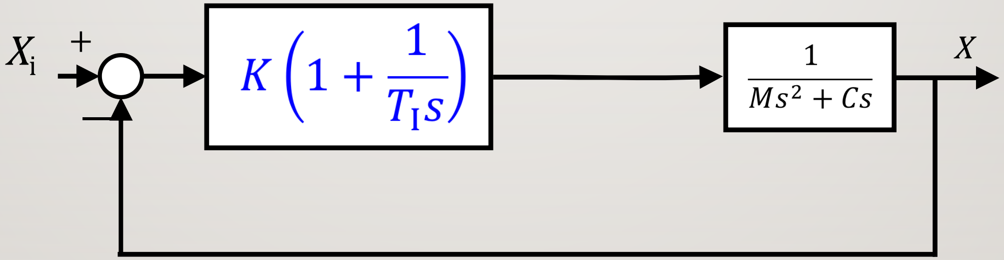


Governing equation:

$$X(s)\left(1+\left(Ks+\frac{K}{T_i}\right)\left(\frac{1}{Ms^3+Cs^2}\right)\right)=X_i(s)\left(Ks+\frac{K}{T_i}\right)\left(\frac{1}{Ms^3+Cs^2}\right)$$

$$X(s)\left(Ms^3+Cs^2+Ks+\frac{K}{T_i}\right)=X_i(s)\left(Ks+\frac{K}{T_i}\right)$$

Proportional and Integral Control (P+I)

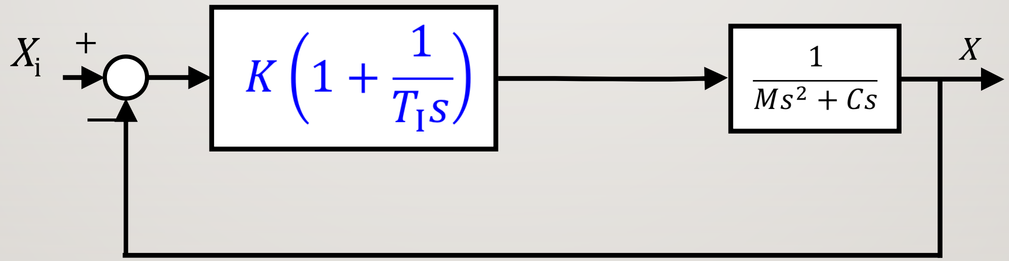


Step input:

$$X(s)\left(Ms^3 + Cs^2 + Ks + \frac{K}{T_i}\right) = \frac{1}{s}\left(Ks + \frac{K}{T_i}\right)$$

$$E(s) = X_i(s) - X(s) = \frac{1}{s}\left(1 - \frac{Ks + \frac{K}{T_i}}{Ms^3 + Cs^2 + Ks + \frac{K}{T_i}}\right)$$

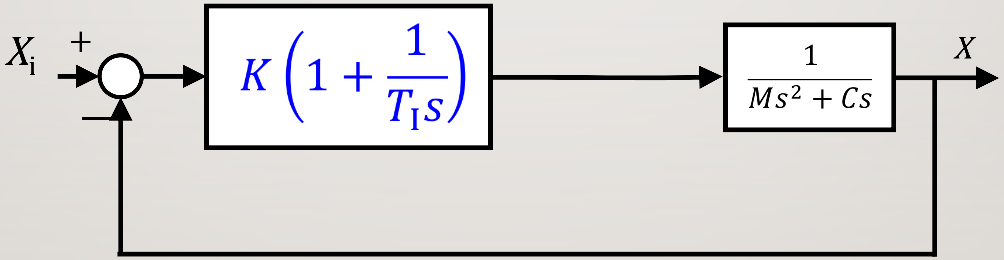
Proportional and Integral Control (P+I)



Final value Theorem for step input:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \left(1 - \frac{Ks + \frac{K}{T_i}}{Ms^3 + Cs^2 + Ks + \frac{K}{T_i}}\right) = 1 - \left(\frac{\frac{K}{T_i}}{\frac{K}{T_i}}\right) = 0$$

Proportional and Integral Control (P+I)

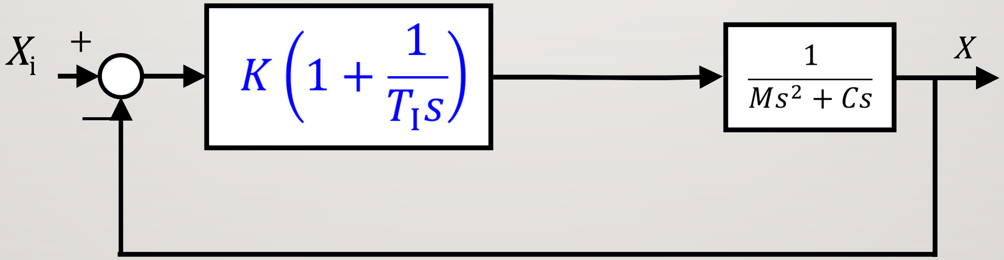


Ramp input:

$$X(s)\left(Ms^3+Cs^2+Ks+\frac{K}{T_i}\right)=\frac{\Omega}{s^2}\left(Ks+\frac{K}{T_i}\right)$$

$$E(s)=X_i(s)-X(s)=\frac{\Omega}{s^2}\left(1-\frac{Ks+\frac{K}{T_i}}{Ms^3+Cs^2+Ks+\frac{K}{T_i}}\right)$$

Proportional and Integral Control (P+I)



Final value Theorem for ramp input:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{\Omega}{s}\left(1 - \frac{Ks + \frac{K}{T_i}}{Ms^3 + Cs^2 + Ks + \frac{K}{T_i}}\right) = \frac{\Omega}{s}\left(1 - \frac{\frac{K}{T_i}}{\frac{K}{T_i}}\right) = 0$$

19 TRANSIENT RESPONSE – THIRD AND HIGHER ORDER SYSTEMS

- Generalised transfer function for the system:

$$G(s) = \frac{Q(s)}{P(s)}$$

$$\mathrm{G}(s) = \frac{Q(s)}{(s-p_1)(s-p_s)\ldots(s-p_N)}$$

20 ROUTH-HURWITZ STABILITY CRITERIA

$$P(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

Routh Hurwitz criteria for stability:

i) Necessary: All coefficients $a_0, a_1, a_2, \ldots, a_n$ are non-zero and have the same sign.

ii) Necessary and sufficient: if i) is satisfied, then the Hurwitz determinants $D_1, D_2, \ldots, D_n$ must be positive.

21 ROUTH-HURWITZ STABILITY CRITERIA (ROUTH ARRAY)

s^n	a_0	a_2	a_4	a_6	...
s^{n-1}	a_1	a_3	a_5	a_7	...
s^{n-2}	b_1	b_2	b_3	...	...
s^{n-3}	c_1	c_2	c_3	...	...
...	...	...	...	...	...
s^0	...	...	...	...	...

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{\mathrm{b}_1}$$

EXAMPLE SHEET 5 QUESTION 4

The transfer function of a control system is as follows:

$$G(s) = \frac{1}{s^3 + 5s^2 + 20s + 6}$$

Is the system stable?

Routh-Hurwitz Array			
s^3			**0**
s^2			0
s	b_1	b_2	b_3
s^0	c_1	c_2	c_3

EXAMPLE SHEET 5 QUESTION 4

The transfer function of a control system is as follows:

$$G(s) = \frac{1}{s^3 + 5s^2 + 20s + 6}$$

Is the system stable?

Routh-Hurwitz Array			
s^3	$\mathbf{1}$	$\mathbf{20}$	$\mathbf{0}$
s^2	5	6	0
s	$b_1 = \frac{5 \times 20 - 1 \times 6}{5}$	$b_2 = \frac{5 \times 0 - 1 \times 0}{5}$	b_3
s^0	c_1	c_2	c_3

EXAMPLE SHEET 5 QUESTION 4

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Is the system stable?

Routh-Hurwitz Array			
s^3	**1**	**20**	**0**
s^2	5	6	0
s	$\frac{94}{5} = 18.8$	0	0
s^0	$c_1 = \frac{18.8 \times 6 - 0}{18.8} = 6$	0	0

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Is the system stable?

Routh-Hurwitz Array			
s^3	**1**	**20**	**0**
s^2	5	6	0
s	18.8	0	0
s^0	6	0	0

No change of sign in the first column: system is stable.

EXAMPLE SHEET 5 QUESTION 4

The transfer function of a control system is as follows:

$$G(s) = \frac{1}{s^3 + 5s^2 + 20s + 6}$$

Use the final value theorem to calculate the unit step response of the system.

$$X(s) = \frac{1}{s} G(s) = \frac{1}{s(s^3 + 5s^2 + 20s + 6)}$$

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s) = s\frac{1}{s} G(s) = \frac{1}{s^3 + 5s^2 + 20s + 6} = \frac{1}{6}$$

FINIS

Any questions?