MMME2046 Dynamics and Control: Lecture 3

Kinematic Analysis of Linkage Mechanism

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Handouts Chapter II

Lecture objectives

- define an instantaneous centre of rotation and use it for velocity analysis
- use equality of velocity projections on the axis joining points for velocity analysis
- perform velocity and acceleration analysis on linkage mechanisms

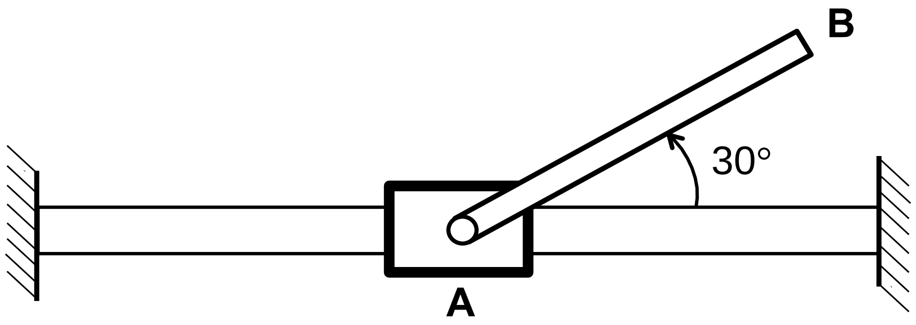
Mechanisms

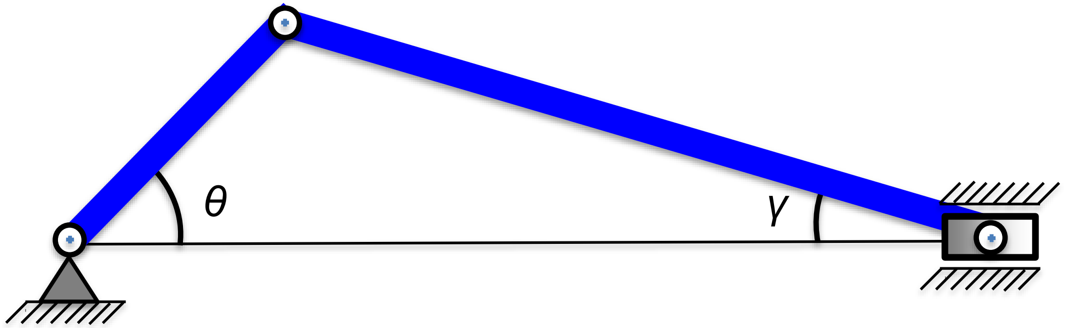
"*Mechanism, […], the means employed to transmit and modify motion in a machine […]. The chief characteristic […] is that all members have constrained motion*" (Encyclopaedia Britannica)

Linkages

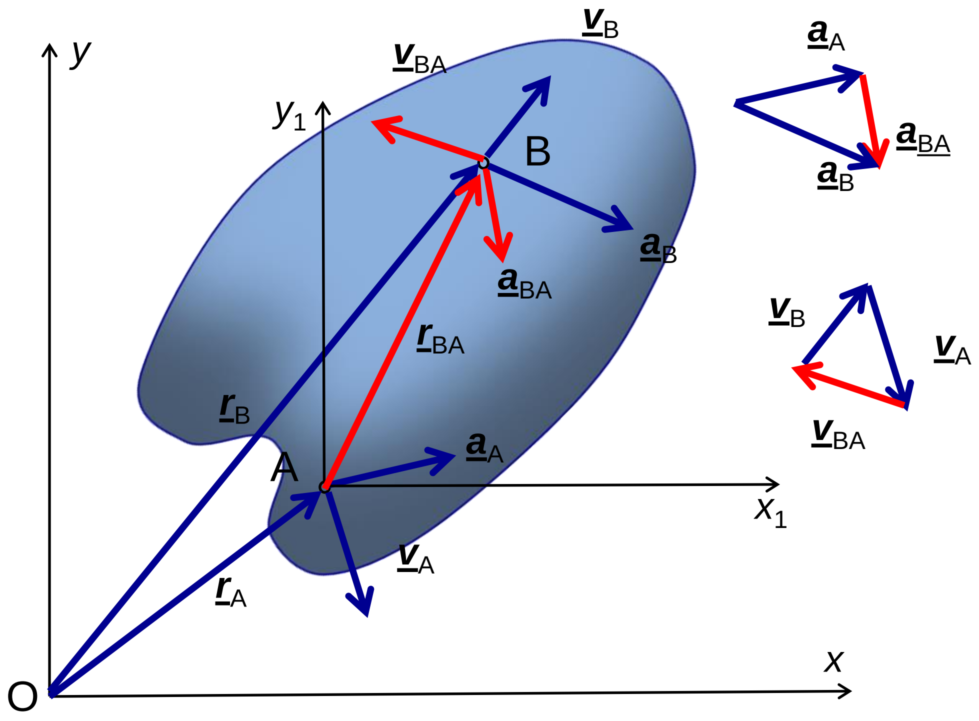
- Linkage is a mechanism that consists of rigid links and one of the links is rigidly attached to a base (frame)

- Constraints are imposed on the rigid body motion





Relative motion



$$\underline{r}_B = \underline{r}_A + \underline{r}_{BA}$$

Fundamental equations for rigid bodies!

$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

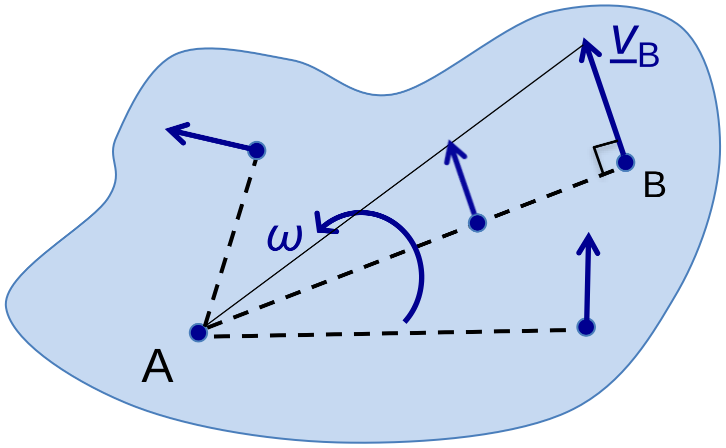
$$\underline{a}_B = \underline{a}_A + \underline{a}_{BA}$$

Note: changing order in “BA” completely changes the physical meaning!

Note: $\underline{r}_{BA}$ is read: ‘Position of B as seen by A’ (A is the reference point)

Instantaneous Centre of Rotation

This is a point with **zero velocity** at a particular moment.

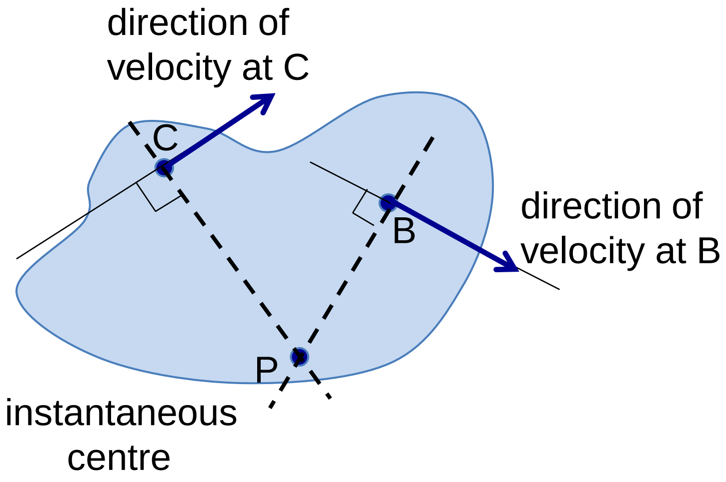


Point A is stationary at instant (instant centre, or velocity pole)

$$\underline{v}_A = 0$$

For any other points: $\underline{v}_B = \underline{v}_{BA}$

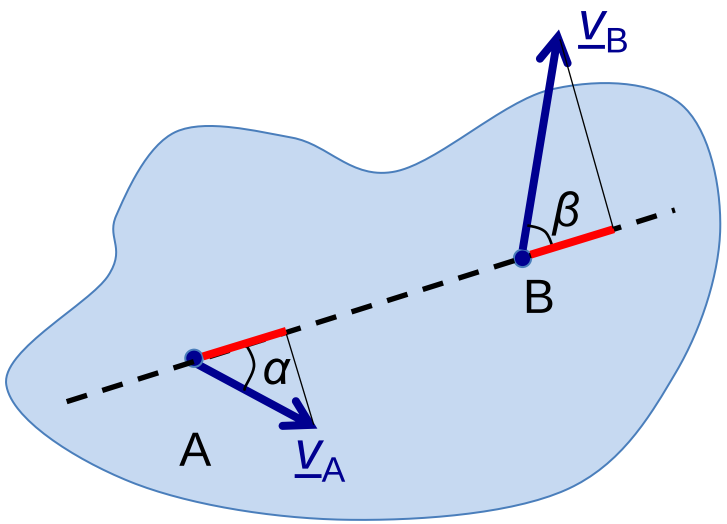
Procedure for finding the centre in most cases



Angular velocity is independent of the choice of origin!

Point velocity projections on joining axis

Take two points A and B & their velocities at one instant



Known: $\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$ (1)

then $\underline{v}_B \parallel AB = \underline{v}_A \parallel AB + \underline{v}_{BA} \parallel AB$

but $\underline{v}_{BA} \parallel AB \equiv 0$ (since $\underline{v}_{BA} \perp AB$)

or $\underline{v}_B \parallel AB = \underline{v}_A \parallel AB$

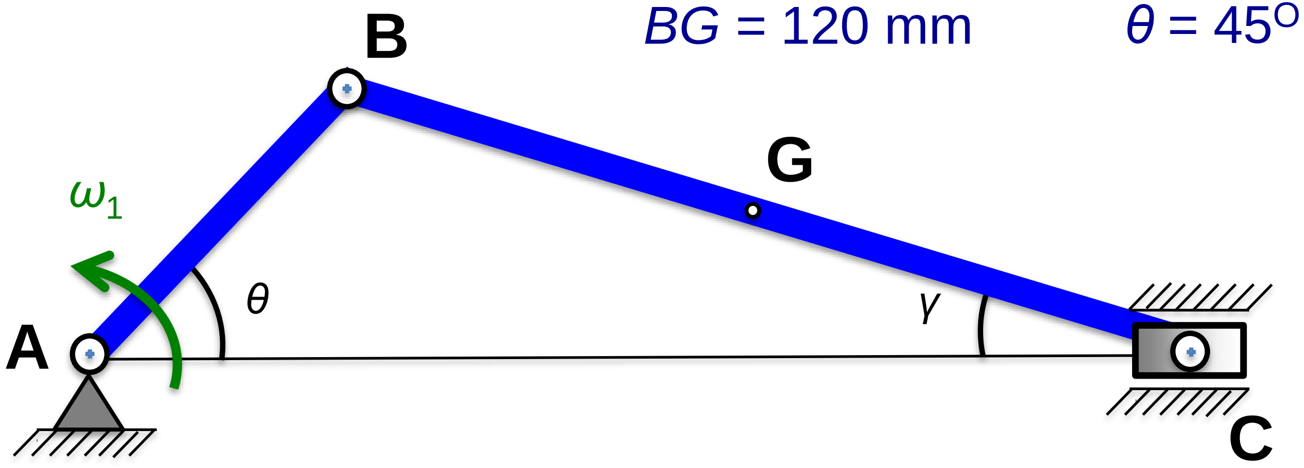
$\underline{v}_B \cos\beta = \underline{v}_A \cos\alpha$ (3)

The velocity components on the axis joining the points are equal.

Worked Example II.4: Crank-Slider mechanism

ω_1 = 100 rad/s = const. AB = 80 mm BC = 240 mm

BG = 120 mm $\theta = 45^{O}$



Geometry:

$$\frac{\sin\gamma}{AB} = \frac{\sin\theta}{BC}, \text{or } \gamma = 13.63°$$

$$AC = \frac{BC\sin 121.4^{\text{o}}}{\sin 45^{\text{o}}} = 0.2897 \text{ m}$$

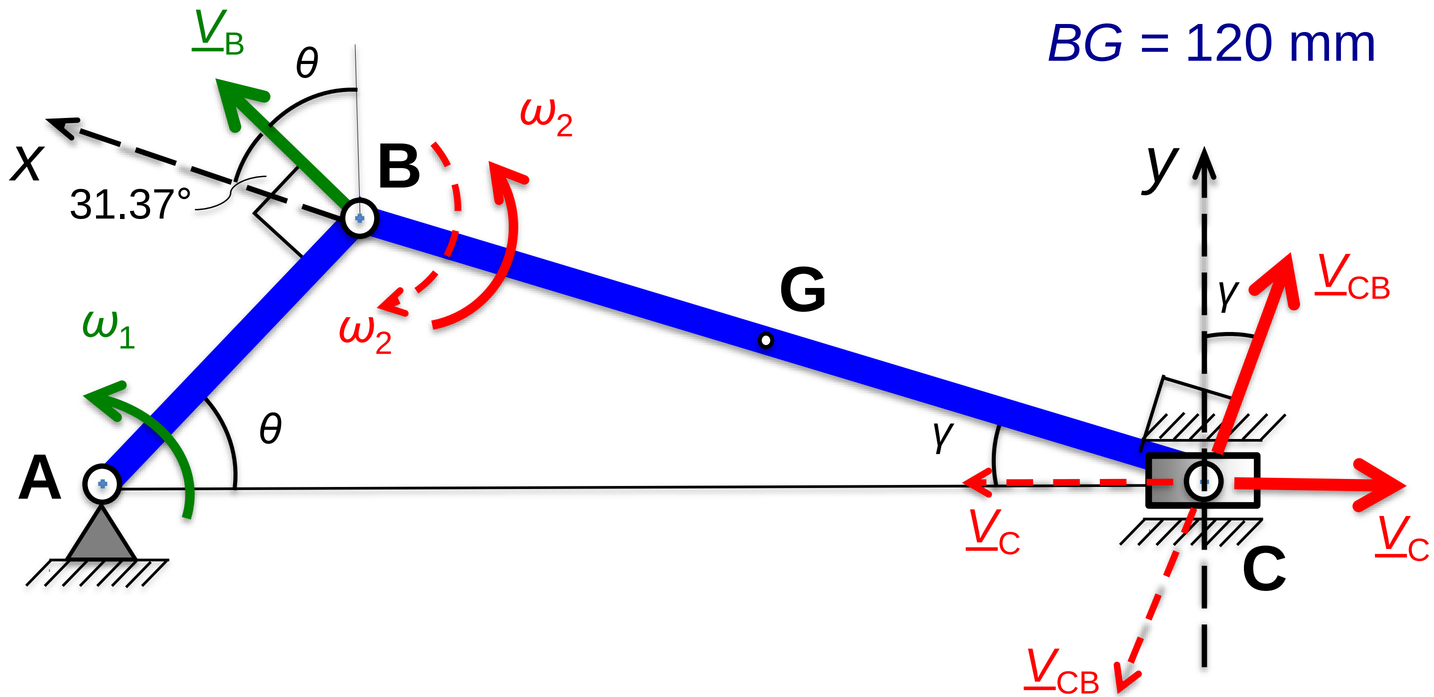
Check: $AC^2 = AB^2 + BC^2 - 2AB.BC\cos 121.4^{\text{o}}$

Worked Example II.4: Crank-Slider mechanism

Velocity Analysis: Method 1

ω_1 = 100 rad/s = const. AB = 80 mm BC = 240 mm

BG = 120 mm γ = 13.63° θ = 45°



$$\mathrm{v}_B = \omega_1 AB = 100 \times 0.08 = 8\ \mathrm{m/s}$$

$$\underline{\mathrm{v}}_C = \underline{\mathrm{v}}_B + \underline{\mathrm{v}}_{CB}$$

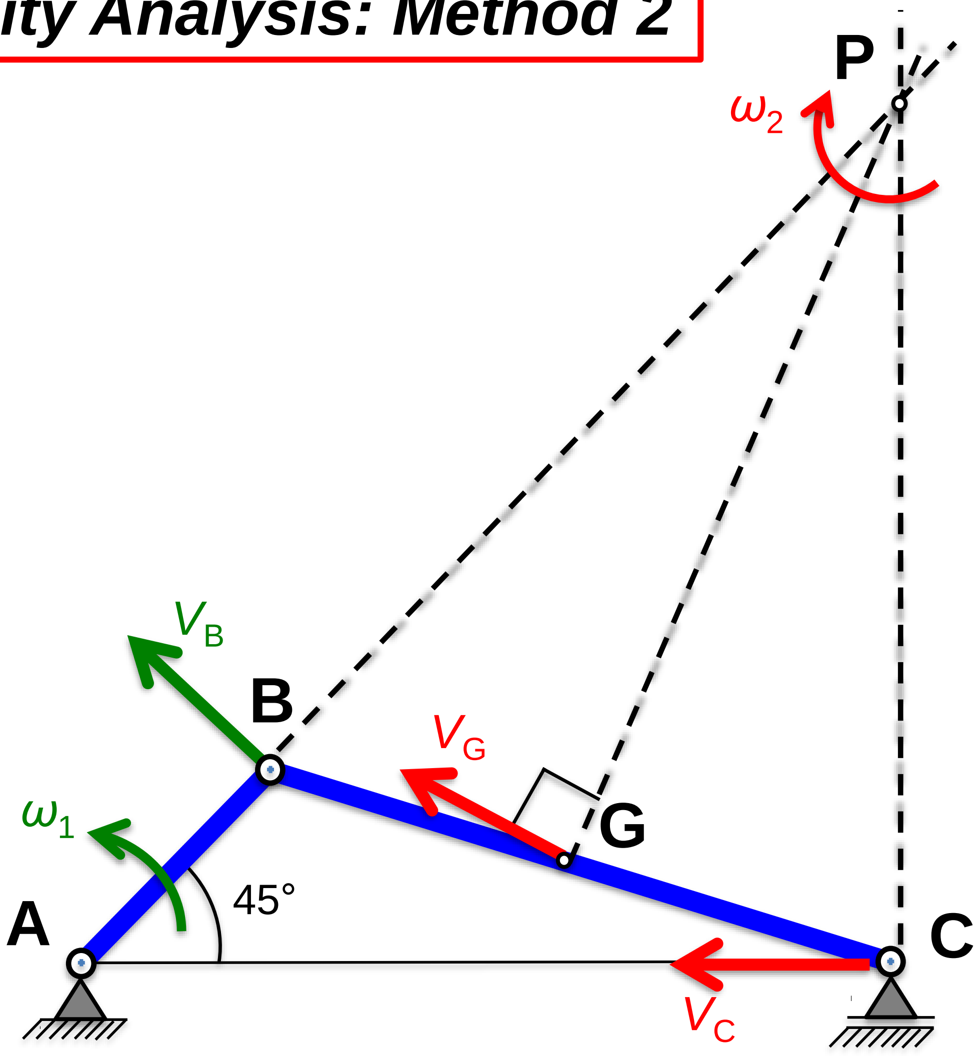
$$\mathrm{v}_{CB} = \omega_2 BC$$

$\rightarrow^{+}\ (\perp Y)$: $\omega_2 =$

$\nwarrow^{+}\ (X)$: $\mathrm{v}_C =$

Worked Example II.4: Crank-Slider mechanism

Velocity Analysis: Method 2



$$v_B = \omega_1 AB = 100 \times 0.08 = 8\ \mathrm{m/s}$$

$$PA = \frac{AC}{\cos 45^o} = 0.4097\ \mathrm{m}$$

$$PB = 0.3297\ \mathrm{m}$$

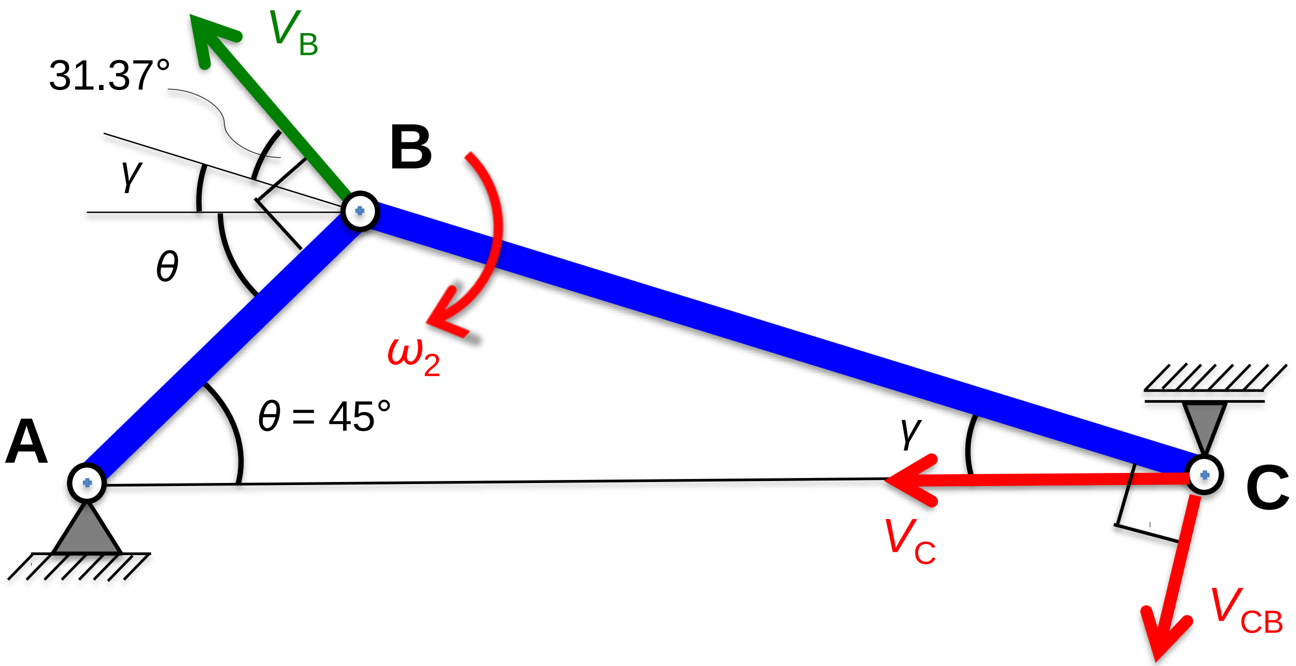
$$\omega_2 =$$

$$PC = \frac{AC}{\tan 45^o} = 0.2897\ \mathrm{m}$$

$$v_C =$$

Worked Example II.4: Crank-Slider mechanism

Velocity Analysis: Method 3 - using Eq.(3)



$$\mathrm{v}_C \cos\gamma = \mathrm{v}_B \cos(90° - \gamma - \theta)$$

$$\mathrm{v}_C \cos 13.63° = \mathrm{v}_B \cos 31.37°$$

$$\mathrm{v}_C =$$

as before

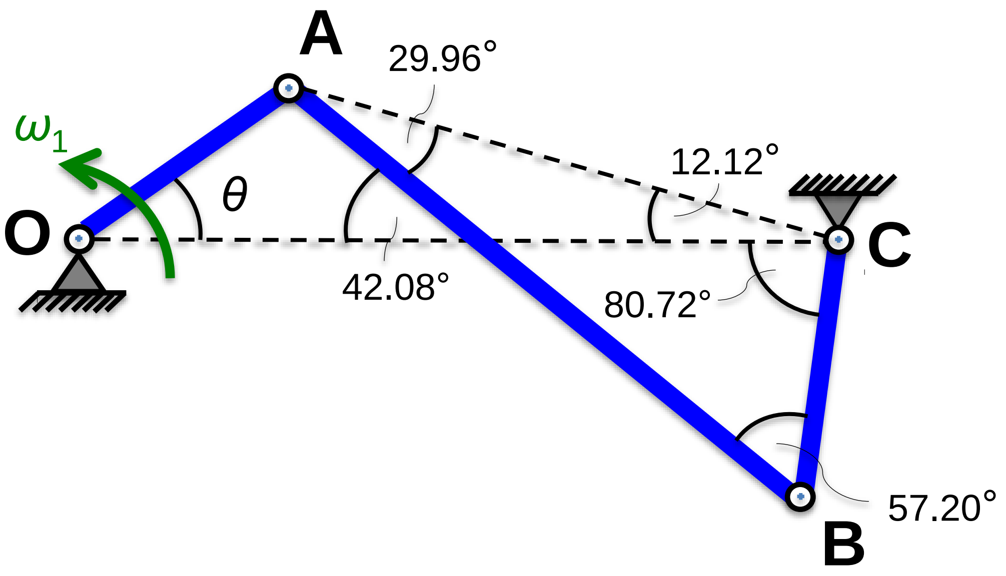
$$\underline{\mathrm{v}}_C = \underline{\mathrm{v}}_B + \underline{\mathrm{v}}_{CB}$$

$\swarrow^{+}$ $(\perp BC)$: $\quad \mathrm{v}_c \sin 13.63° = -\mathrm{v}_B \sin 31.37° + \omega_2 BC$

$$\omega_2 = 24.25 \text{ rad/s}$$

Worked Example II.5: Four-bar linkage mechanism

"Base"
(frame)



(http://www.mrbillington.com/linkages.html)

Worked Example II.5: Four-bar linkage mechanism

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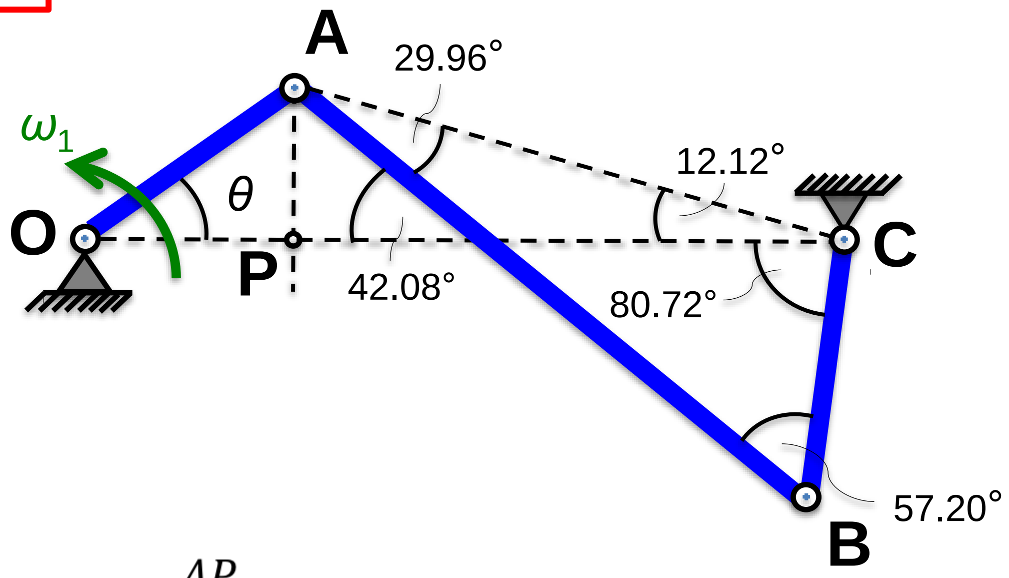
Geometry



OA = 30 mm

BC = 60 mm

AB = 120 mm

OC = 120 mm

$\theta = 45^{O}$

ω_1 = 30 rad/s = const.

$$OP = AP = OA\cos 45^{o} = 21.21 \text{ mm}$$

$$PC = OC - OP = 98.79 \text{ mm}$$

$$AC = \sqrt{PC^2 + AP^2} = 101.0 \text{ mm}$$

$$\angle \text{PCA} = \tan^{-1}\frac{AP}{PC} = 12.12°$$

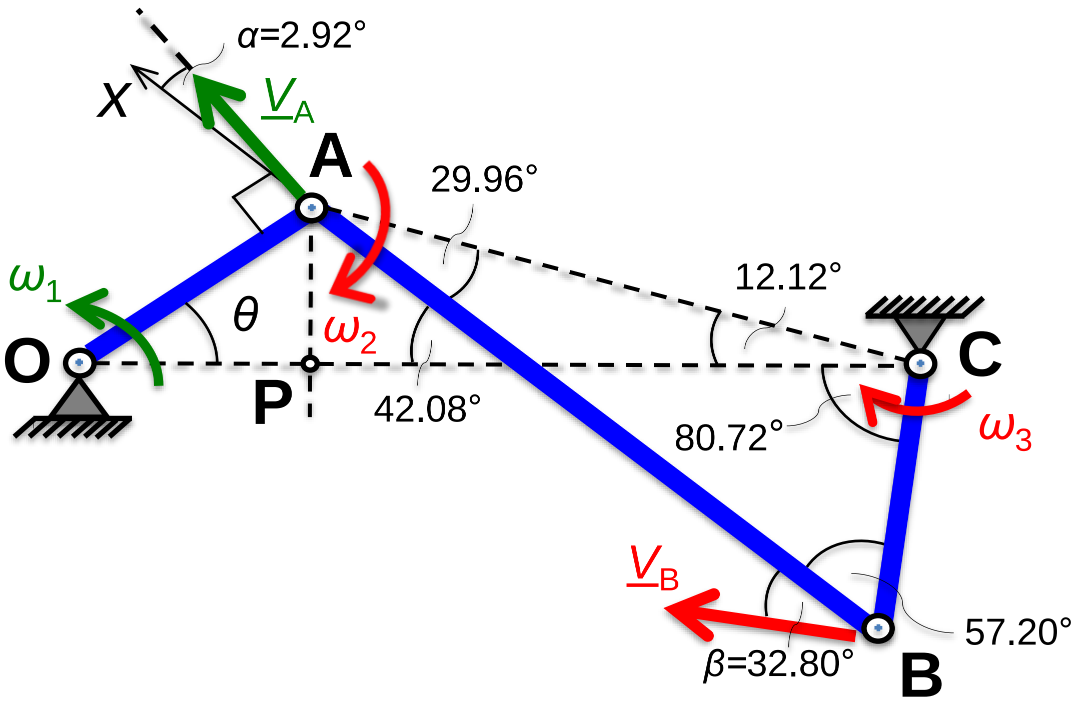
$$\angle ABC = \sin^{-1}\left(\frac{AC}{BC}\sin 29.96°\right) = 57.20°$$

$$\angle \text{BAC} = \cos^{-1}\frac{AB^2 + AC^2 - BC^2}{2AB.AC} = 29.96°$$

$$\angle PCB = 80.72°$$

Worked Example II.5: Four-bar linkage mechanism

Velocity Analysis: Using Eq.(3)



$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

$\swarrow^{+}$ $(\perp AB)$:

$$v_A = \omega_1 OA = 30 \times 0.03 = 0.9 \text{ m/s}$$

Eq. (3) – projecting velocities $\underline{v}_A$ and $\underline{v}_B$ on AB:

$$\omega_2 =$$

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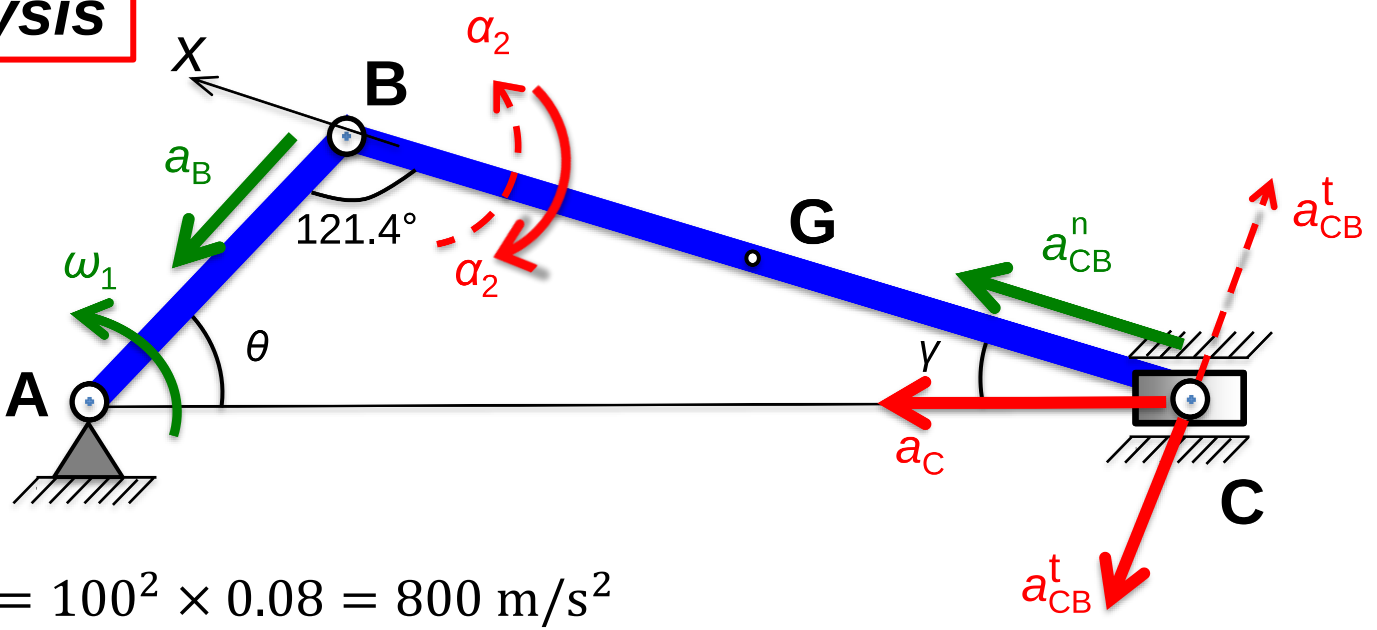
Acceleration Analysis



$$a_B \equiv a^n_B = \omega_1^2\, AB = 100^2 \times 0.08 = 800\ \text{m/s}^2$$

$$\underline{a}_C = \underline{a}_B + \underline{a}^n_{CB} + \underline{a}^t_{CB}$$

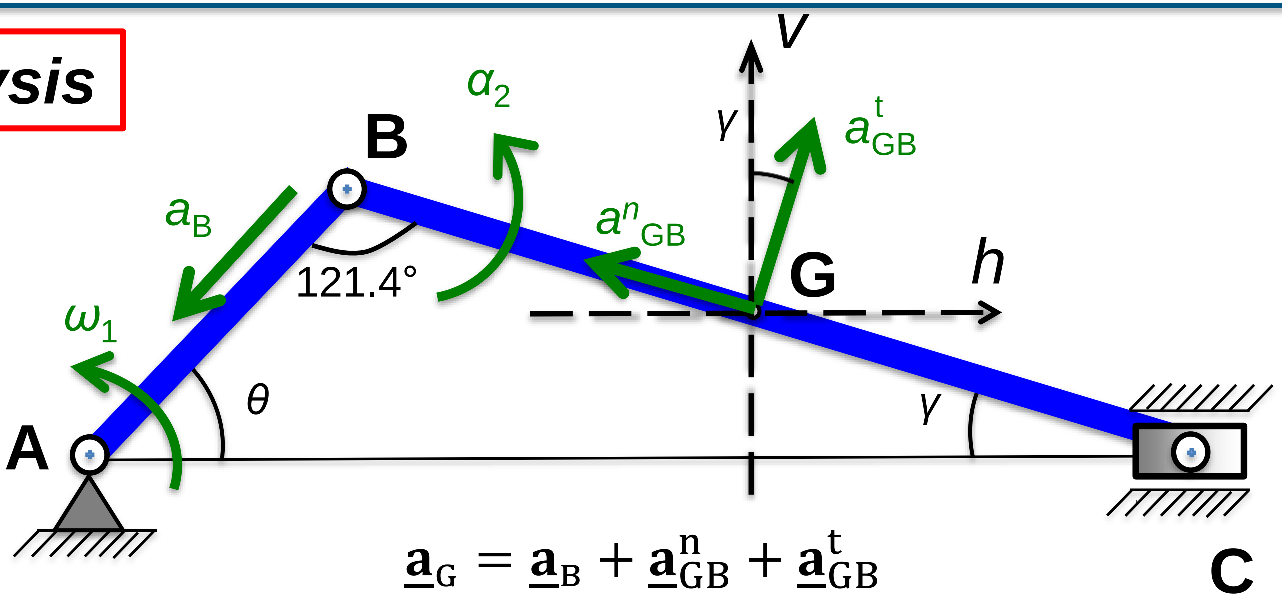
$$a^n_{CB} = \omega_2^2\, BC = 24.25^2 \times 0.24 = 141.1\ \text{m/s}^2$$
$$a^t_{CB} = \alpha_2\, BC = 0.24\alpha_2$$

$\nwarrow^+ \Sigma X$:

$\uparrow^+ \Sigma Y$:

Worked Example II.4: Crank-Slider mechanism (cont.)

Acceleration Analysis



$$\underline{\mathbf{a}}_G = \underline{\mathbf{a}}_B + \underline{\mathbf{a}}_{GB}^{n} + \underline{\mathbf{a}}_{GB}^{t}$$

$$a_{GB}^{n} = \omega_2^2 . BG = 24.25^2 \times 0.12 = 70.55 \text{ m/s}^2$$
$$a_{GB}^{t} = \alpha_2 . BG = 2283 \times 0.12 = 274.0 \text{ m/s}^2$$

$\rightarrow^{+} \Sigma H: \; a_{Gh} =$

$\uparrow^{+} \Sigma V: \; a_{Gv} =$

$$a_G = \sqrt{a_{Gh}^2 + a_{Gv}^2} = \sqrt{(-579.7)^2 + (-282.8)^2} = 636.0 \text{ m/s}^2$$

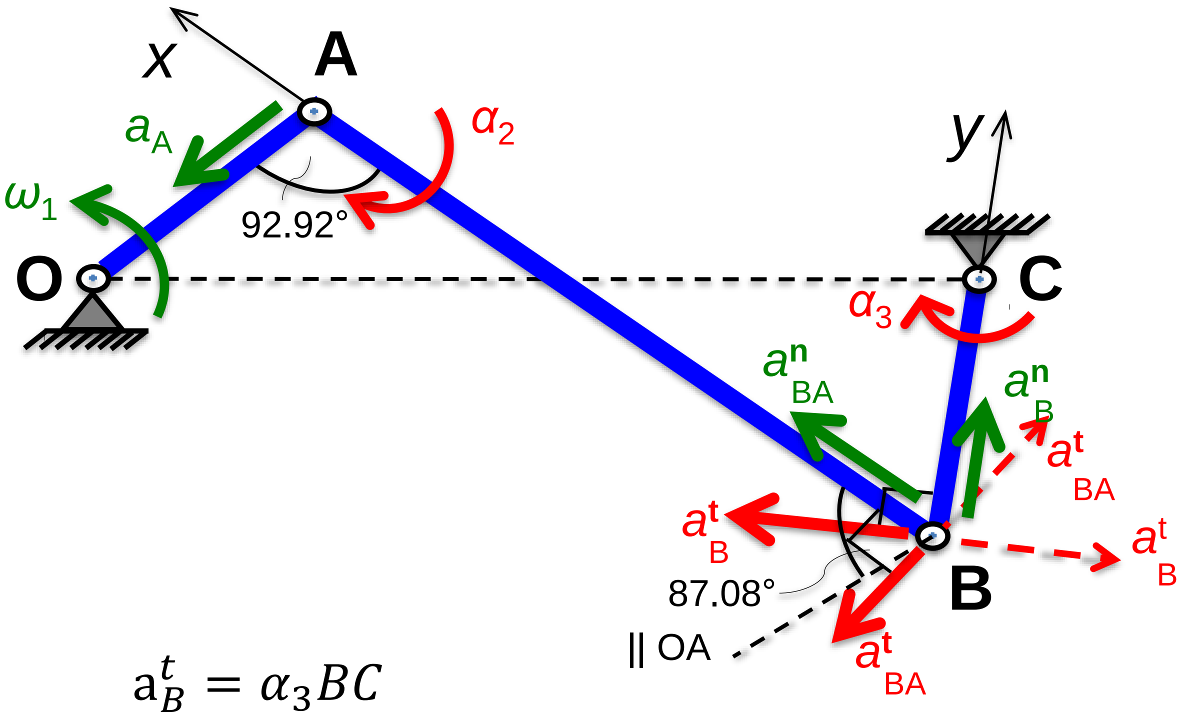
Acceleration Analysis

$$\underline{a}_B = \underline{a}_B^n + \underline{a}_B^t$$

$$\underline{a}_B = \underline{a}_A + \underline{a}_{BA}^n + \underline{a}_{BA}^t$$

$$\underline{a}_B^n + \underline{a}_B^t = \underline{a}_A + \underline{a}_{BA}^n + \underline{a}_{BA}^t \qquad (5)$$

$$a_A = a_A^n = \omega_1^2 OA = 30^2 \times 0.03 = 27\ \mathrm{m/s^2}$$



$$a_B^n = \omega_3^2\, BC = 17.82^2 \times 0.06 = 19.05\ \mathrm{m/s^2} \qquad a_B^t = \alpha_3 BC$$

$$a_{BA}^n = \omega_2^2\, AB = 5.208^2 \times 0.12 = 3.255\ \mathrm{m/s^2} \qquad a_{BA}^t = \alpha_2 AB$$

$\nwarrow^+ \Sigma X$:

$\uparrow^+ \Sigma Y$:

Lecture objectives

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Next lecture

- revise particle dynamics and mass moments of inertia
- define degrees of freedom for rigid body motion
- introduce fundamental relationships for rigid body dynamics
- formulate appropriate equations of motion
- apply planar dynamics to the solution of practical problems