University of Nottingham
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LECTURE 2A

Simple Electrical Circuits

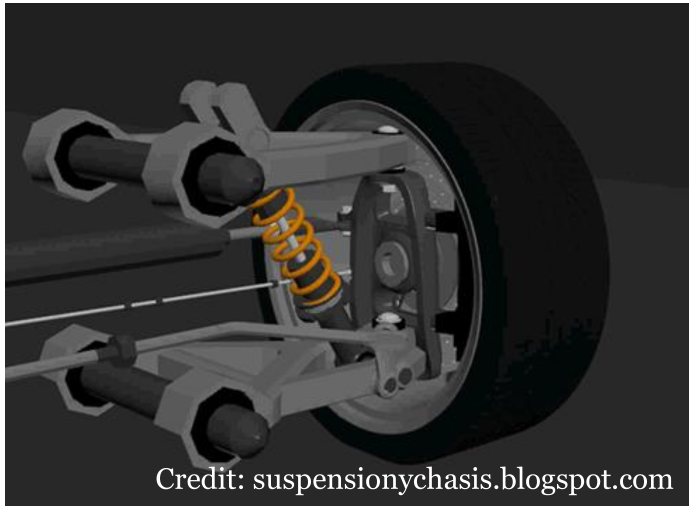
Electromechanical Devices
MMME2051

Module Convenor – Surojit Sen

Learning Outcomes

- **Basic Concepts (& Recap)**
  - Charge, current, voltage
  - Ohm’s/Kirchhoff’s Laws
  - Power & Energy
  - Measurement of Voltage & Current
  - Electrical symbols & notations
  - Solution of a simple electrical circuit using just Ohm’s and Kirchhoff’s Laws
- Electrical circuits
  - **Series** & **Parallel**
  - **Combination** of series & parallel
  - **Example circuits** (to be discussed in upcoming seminar)
- Further Reading

Engineering System



Suspension & **chasses**
Mechanical Engineering

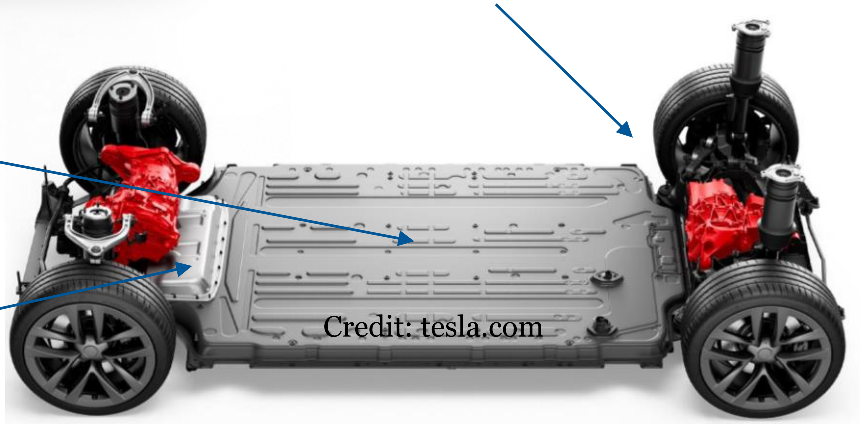
Vehicle Control Unit (VCU) that sends signals/commands to drive/stop the car
Electronic Engineering

Code written to program the VCU
Computer/Software Engineering

Motor that converts electrical power from battery to mechanical motion
Electromechanical Engineering

Battery that supplies power to drive the motor
Electrical Engineering

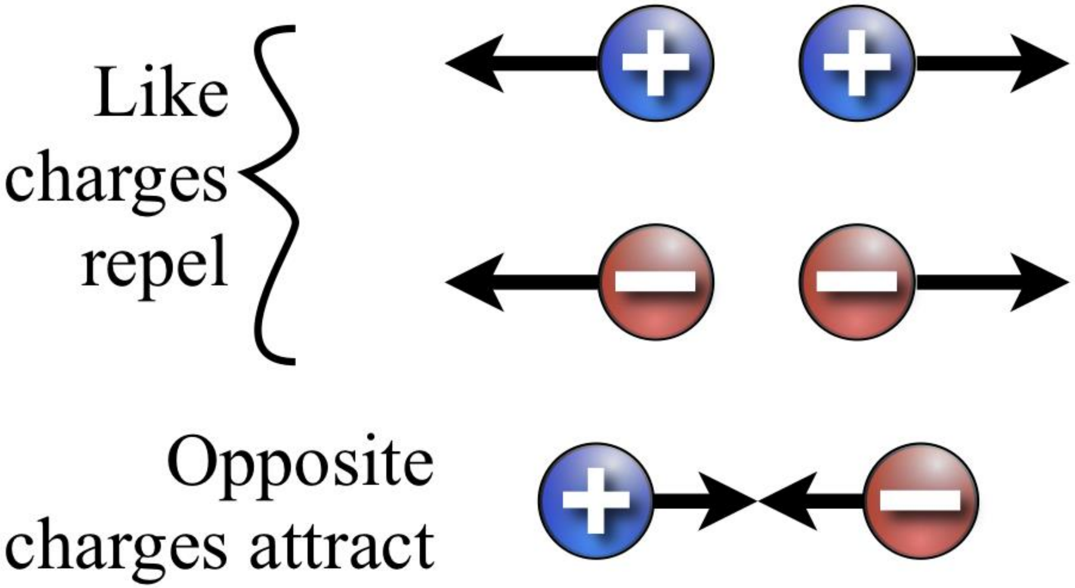
Power Converter that controls the flow of electrical power between battery and motor
Power Electronics



Electric Charge

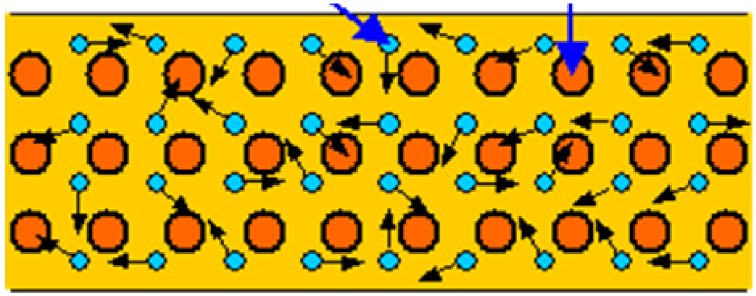
Some important characteristics of electric charge

- Charge can be positive or negative
- Nature's basic carrier of negative charge is the **electron**
- Nature's basic carrier of positive charge is the **proton**
- If an object has a deficit of electrons, it will exhibit a **net positive charge**
- If an object has a surplus of electrons, it will exhibit a **net negative charge**

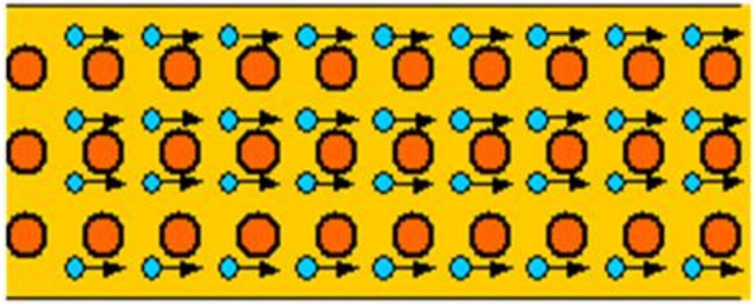


Electric Current

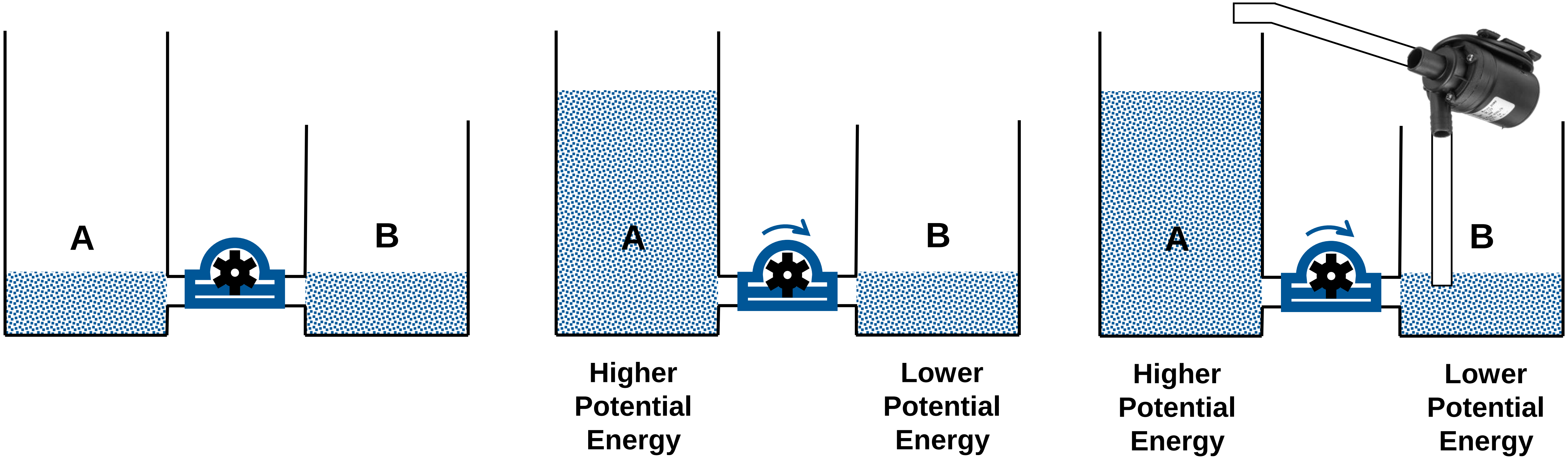
In a conductor, there are many free electrons and they move randomly



If the free electrons move consistently, a electron flow will be seen



Electric Potential (Voltage)



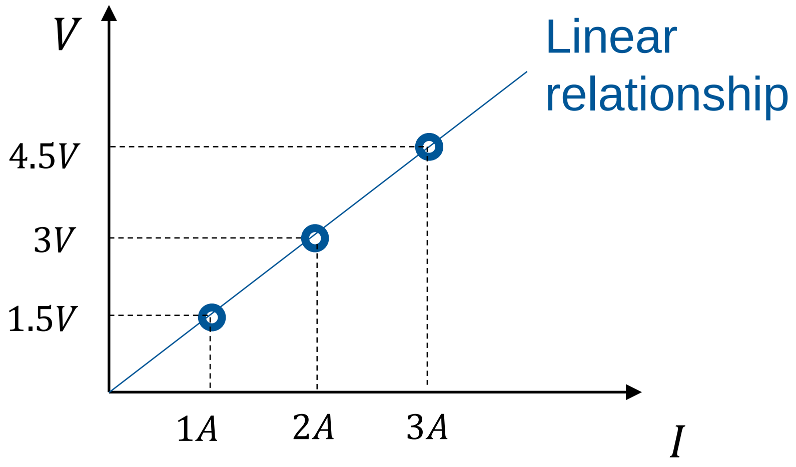
Basic concepts in electrical engineering

Ohm's Law

Voltage is linearly proportional to resistance

$$V = IR$$

Voltage (V), Current (I), Resistance (R)



Power

Product of voltage and current

$$P = VI$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

Unit is **Watt** (W)

Energy

Accumulation of power over time

$$E = P\Delta t$$

$$E = I^2 R \Delta t$$

$$E = \frac{V^2}{R}\Delta t$$

Unit is **Joule** (J)

Kilowatt-hour (kWh) often used

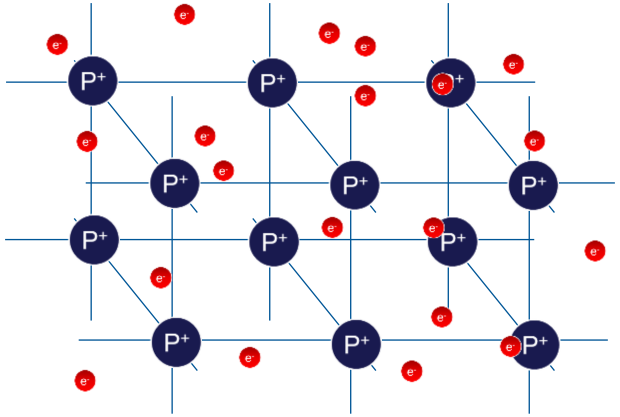
Basic concepts in electrical engineering

Conductor

Easily allows current to flow through it

$$R \rightarrow 0\Omega$$

All metals are conductors

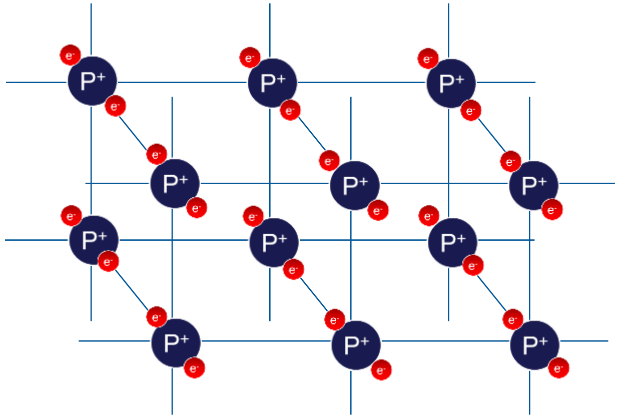


Insulator

Strongly impedes flow of current through it

$$R \rightarrow \infty\Omega$$

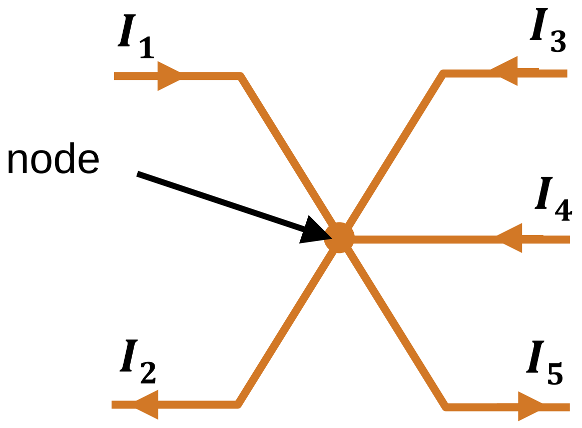
Plastic, rubber, wood etc.



Basic concepts in electrical engineering

Kirchhoff's Current Law

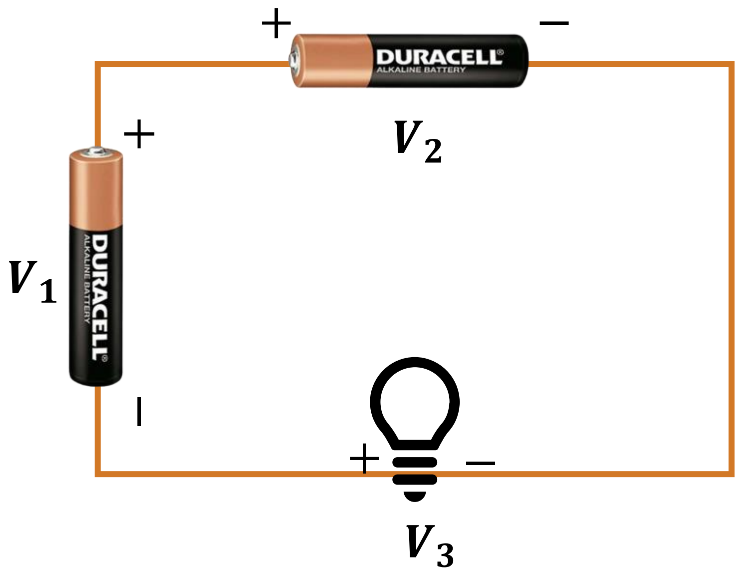
Algebraic sum of current entering a node is zero



$$\sum I_i = 0$$

Kirchhoff's Voltage Law

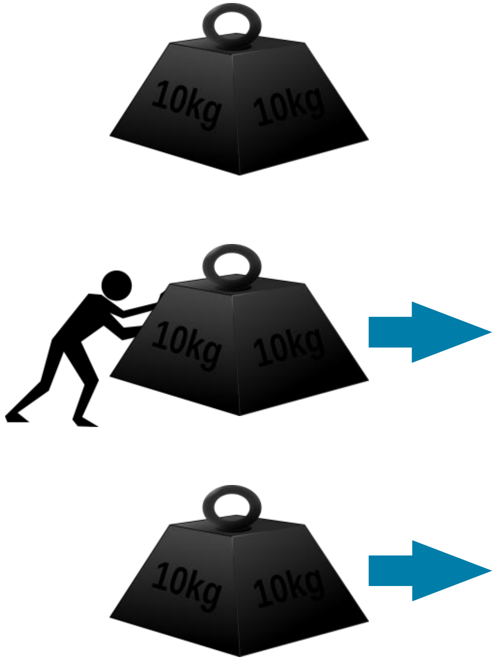
Algebraic sum of voltages around a closed loop is zero

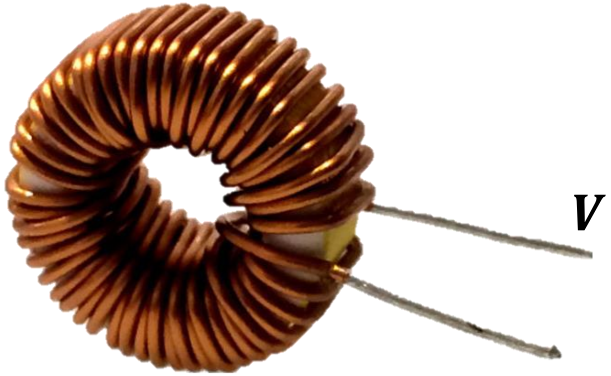


$$\sum V_i = 0$$

Basic concepts in electrical engineering

Reactive elements store energy, and responds/behaves according to the **present AND the past!**





$$V = L\frac{dI}{dt}$$

Inductor **opposes sudden changes** in **current**

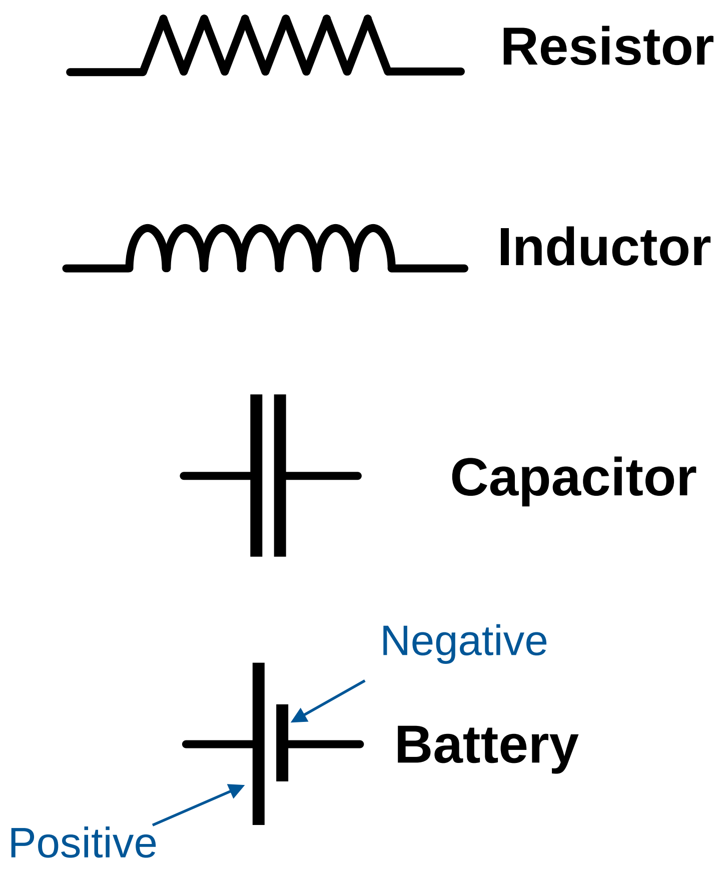
By **inducing** as much **voltage** is theoretically needed to **keep the current steady**

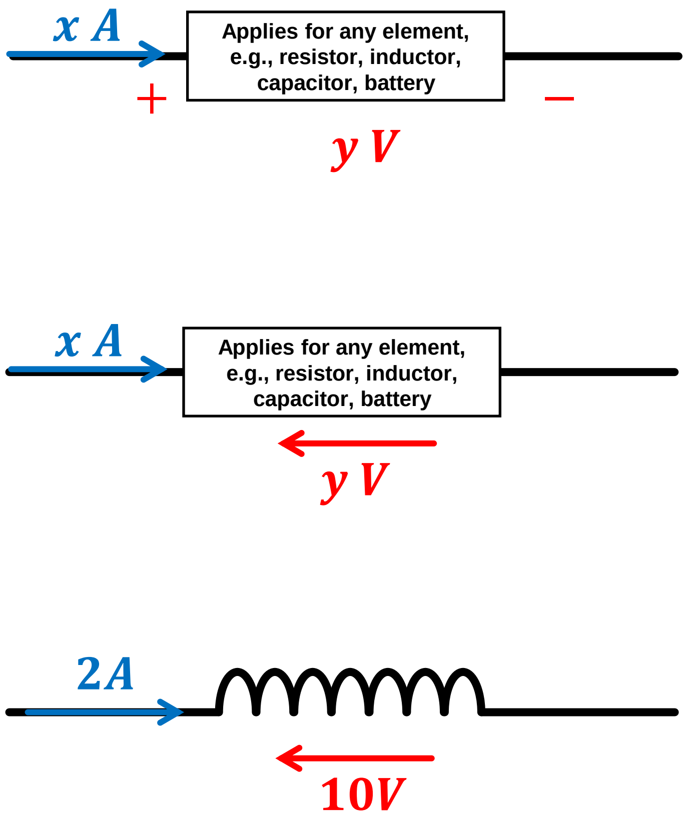
$$I = C\frac{dV}{dt}$$

Capacitor **opposes sudden changes** in **voltage**

By **generating** as much **current** is theoretically needed to **keep the voltage steady**

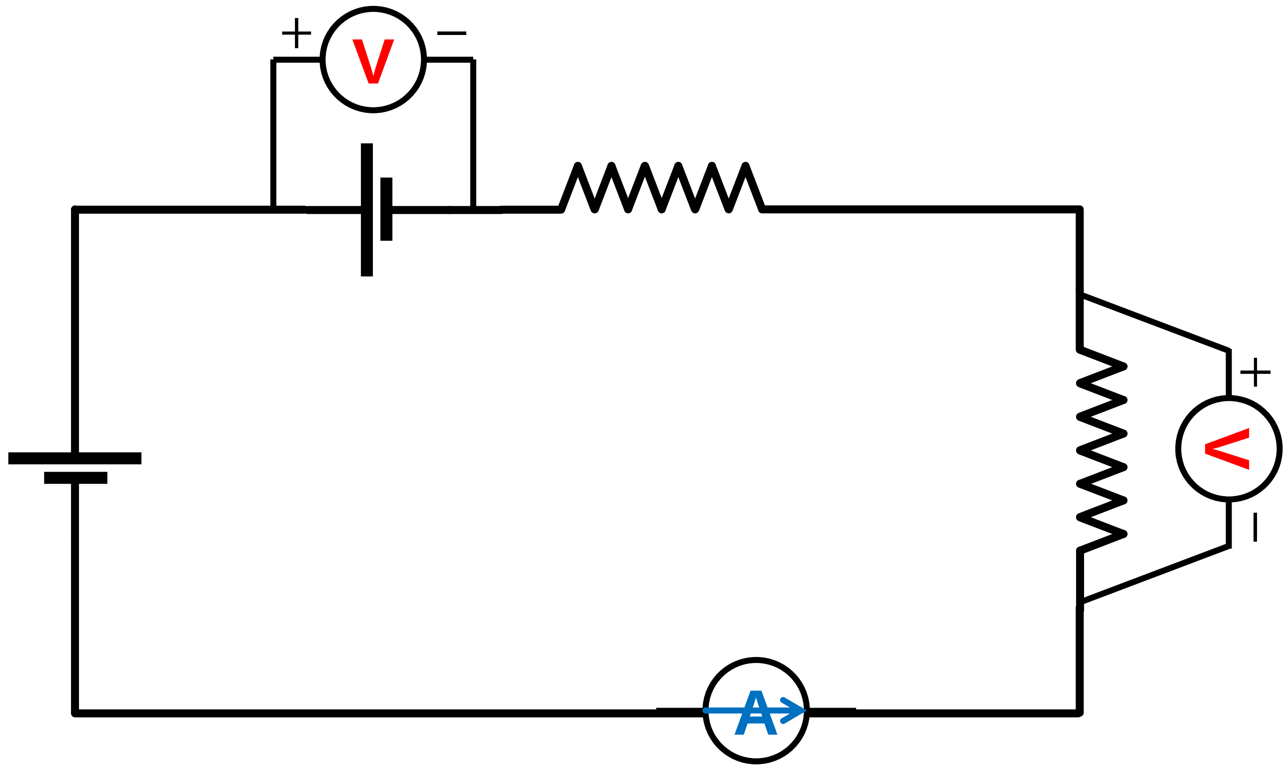
Symbols & Notations used in circuit diagrams





Measurement of Current & Voltage

A physical device that is added to an electrical circuit (that you want to observe) **without affecting** the working of the circuit



Voltmeter is added in **parallel to the element** (or group of elements) to measure the voltage across it

Its internal impedance is **very high**

$$R \rightarrow \infty\Omega$$

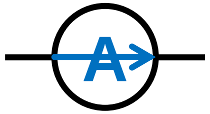
Ammeter is added in **series to the element** (or group of elements) to measure the current through it

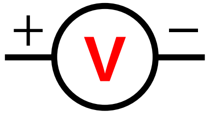
Its internal impedance is **very low**

$$R \rightarrow 0\Omega$$

Measurement of Current & Voltage

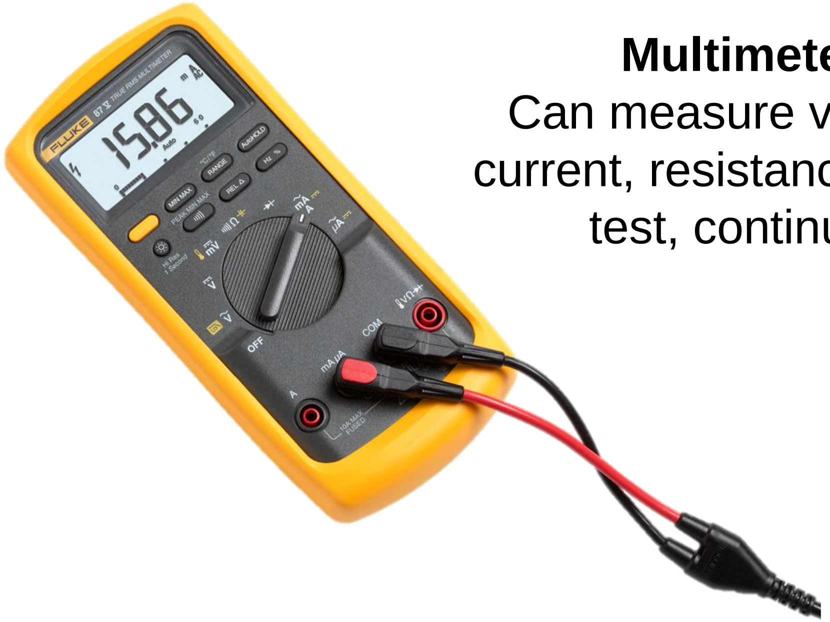
A physical device that is added to an electrical circuit (that you want to observe) **without affecting** the working of the circuit





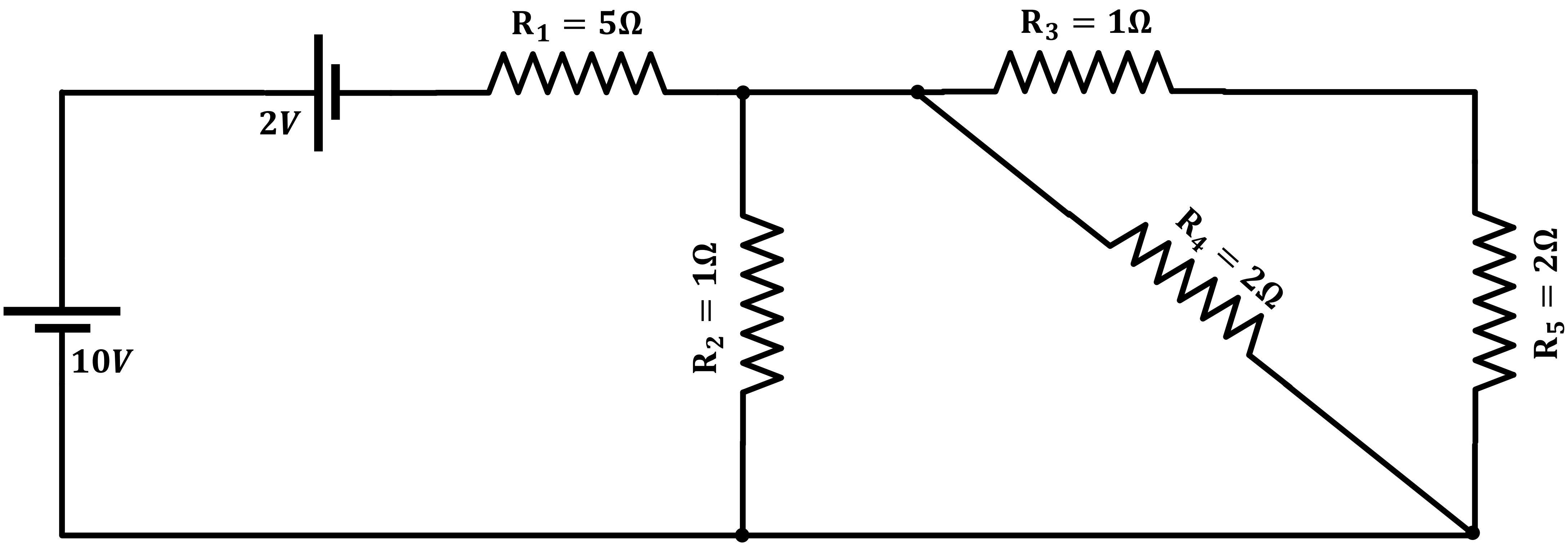
Ammeter

Voltmeter



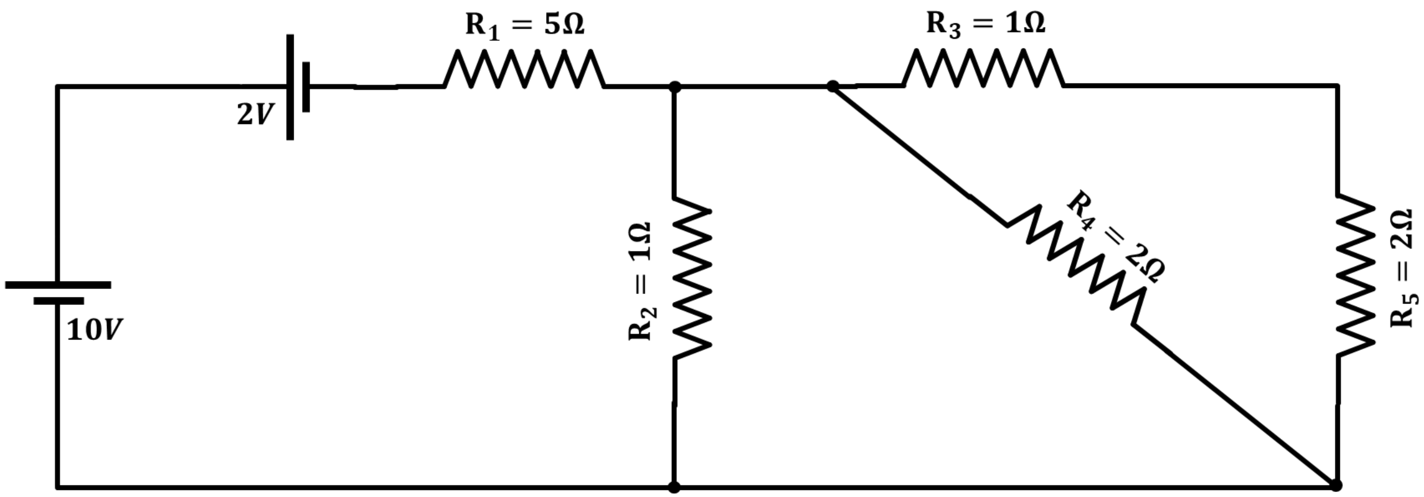
Multimeter
Can measure voltage, current, resistance, diode test, continuity

Application of Kirchhoff's Law



Find out the current and voltage of every resistor

Application of Kirchhoff's Law



Step 1 – Identify all the loops in the circuit

Step 2 – Assign a "loop current" variable

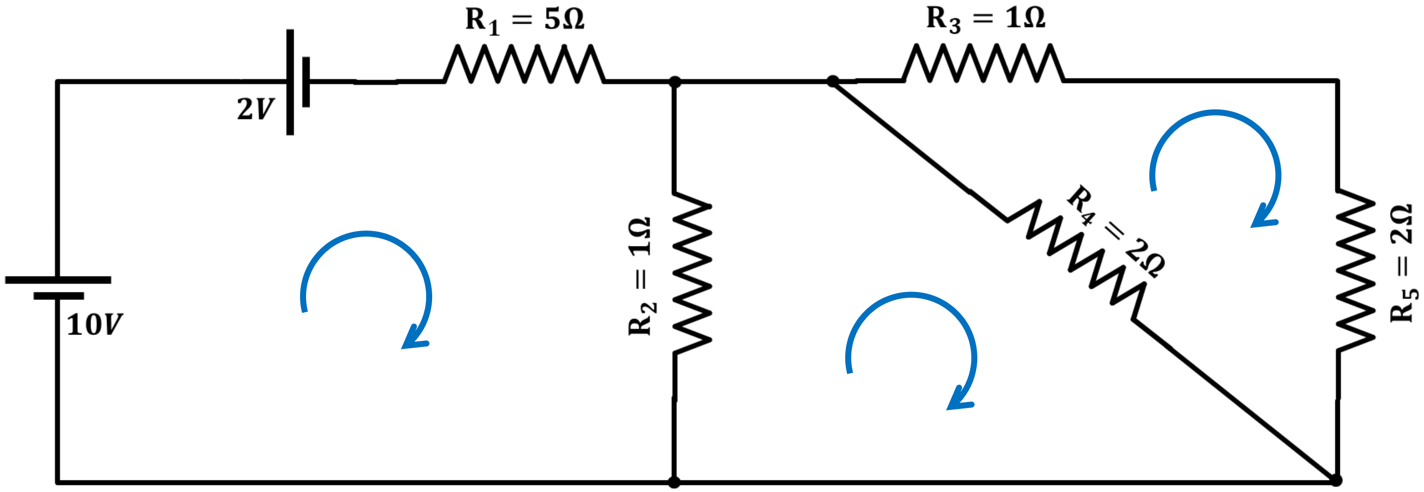
Step 3 – Identify "branch current" values (apply KCL)

Step 4 – Apply KVL to each loop

Step 5 – Apply Ohm's Law

Step 6 – Solve the linear system of equations – you can solve for n unknowns with n equations

Application of Kirchhoff's Law



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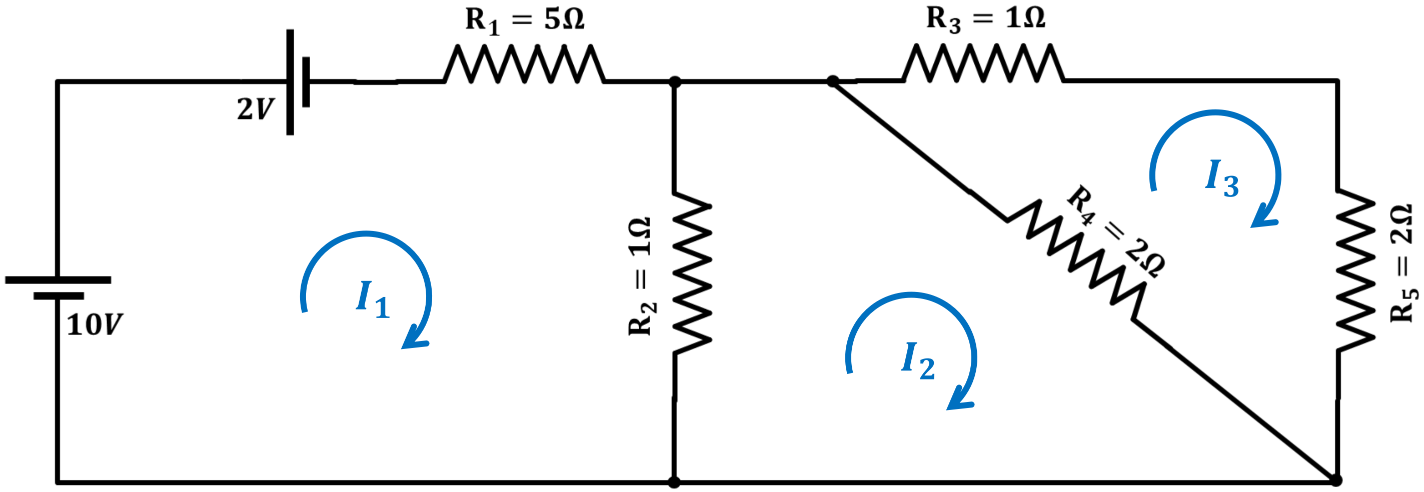

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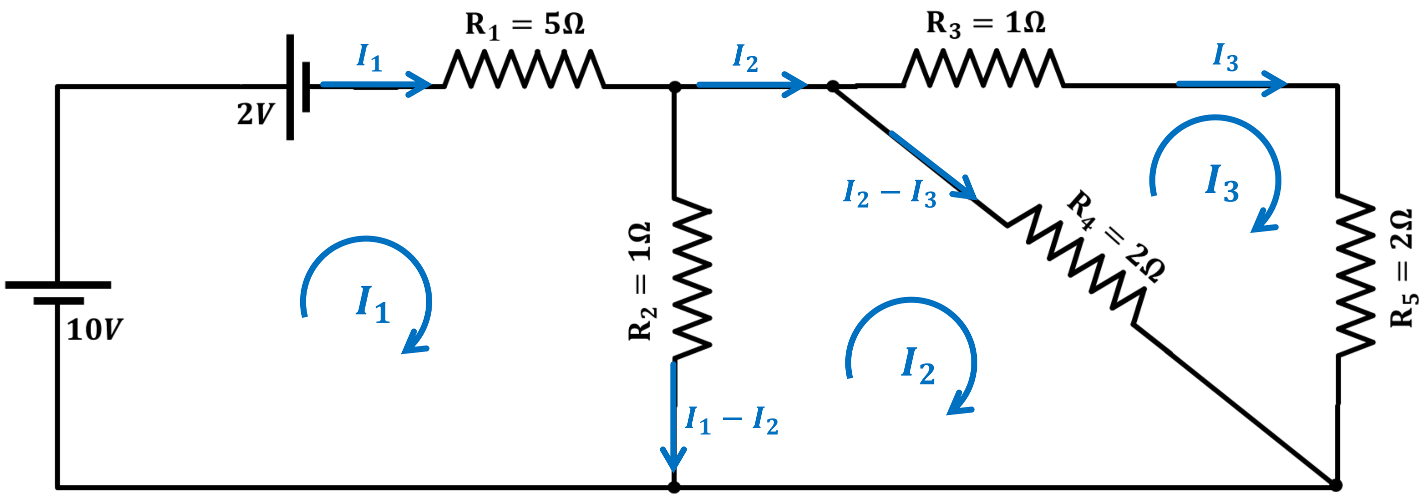

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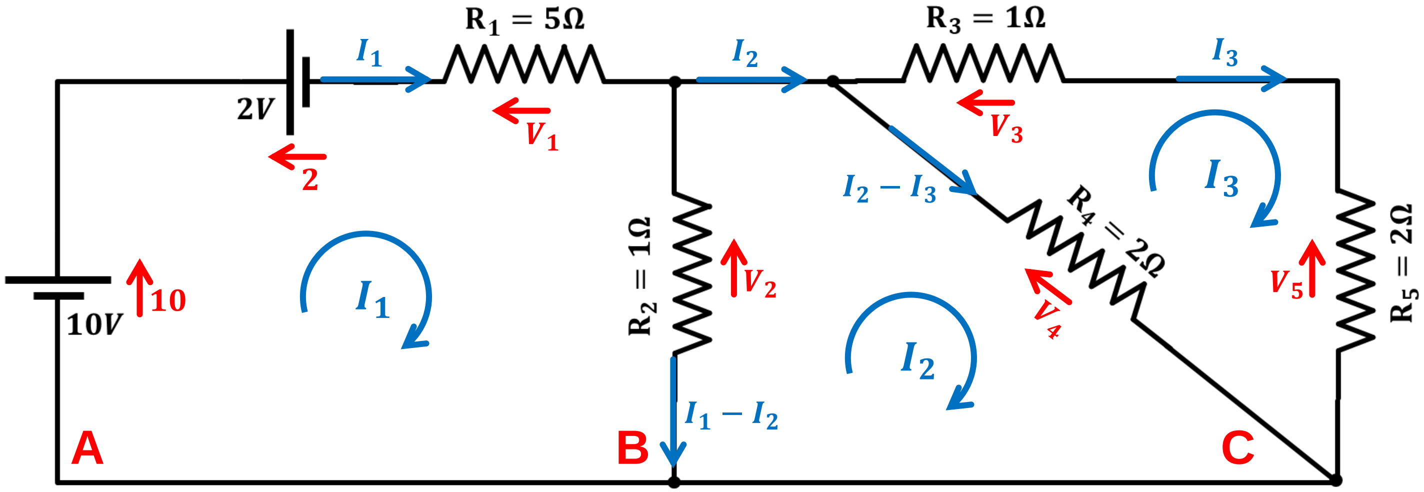
Application of Kirchhoff's Law



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Step 6 – Solve the linear system of equations – you can solve for n unknowns with n equations

Loop 1 KVL (node A origin):

$$10 - 2 - V_1 - V_2 = 0$$

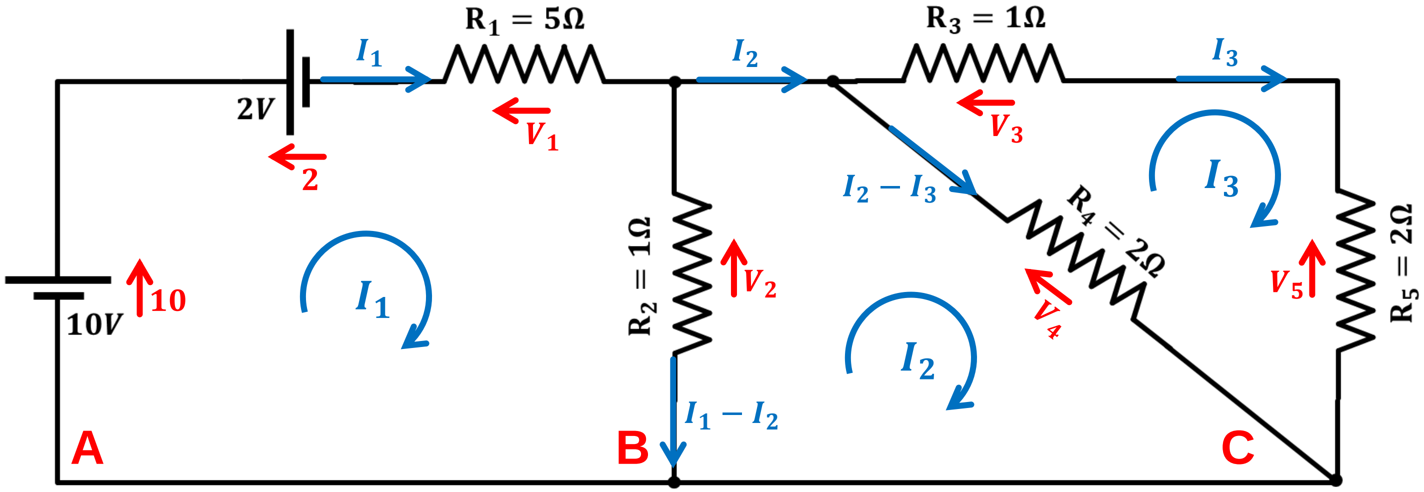
Loop 2 KVL (node B origin):

$$V_2 - V_4 = 0$$

Loop 3 KVL (node C origin):

$$V_4 - V_3 - V_5 = 0$$

Application of Kirchhoff's Law



~~**Step 1 – Identify all the loops in the circuit**~~

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Step 6 – Solve the linear system of equations – you can solve for n unknowns with n equations

Loop 1 KVL (node A origin):

$$10 - 2 - V_1 - V_2 = 0$$
$$8 - I_1R_1 - (I_1 - I_2)R_2 = 0$$
$$I_1(R_1 + R_2) - I_2R_2 = 8$$
$$I_1(5 + 1) - I_2 1 = 8$$
$$\boldsymbol{6I_1 - I_2 = 8}$$

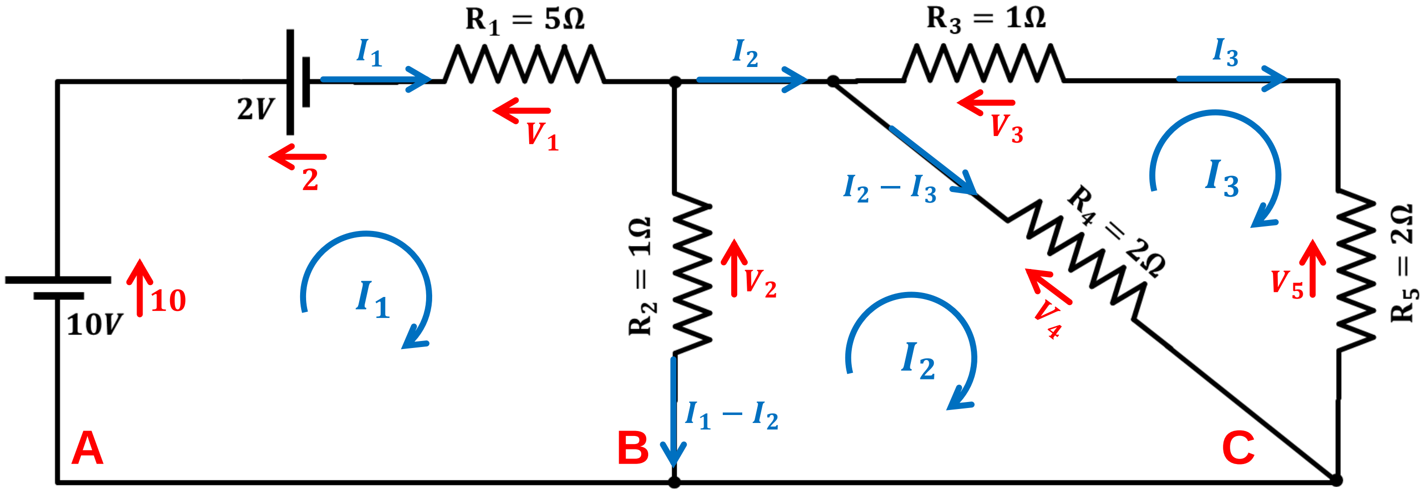
Loop 2 KVL (node B origin):

$$V_2 - V_4 = 0$$
$$(I_1 - I_2)R_2 - (I_2 - I_3)R_4 = 0$$
$$I_1R_2 - I_2\,(R_2 + R_4) + I_3R_4 = 0$$
$$I_1 1 - I_2\,(1 + 2) + I_3 2 = 0$$
$$\boldsymbol{I_1 - 3I_2 + 2I_3 = 0}$$

Loop 3 KVL (node C origin):

$$V_4 - V_3 - V_5 = 0$$
$$(I_2 - I_3)R_4 - I_3R_3 - I_3R_5 = 0$$
$$I_2R_4 - I_3(R_3 + R_4 + R_5) = 0$$
$$I_2 2 - I_3(1 + 2 + 2) = 0$$
$$\boldsymbol{2I_2 - 5I_3 = 0}$$

Application of Kirchhoff's Law



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$$\mathbf{6I_1 - I_2 = 8} \qquad Eq1$$
$$\mathbf{I_1 - 3I_2 + 2I_3 = 0} \qquad Eq2$$
$$\mathbf{2I_2 - 5I_3 = 0} \qquad Eq3$$

Apply $3(Eq1) - (Eq2)$:

$$18I_1 - 3I_2 - I_1 + 3I_2 - 2I_3 = 24$$
$$\mathbf{17I_1 - 2I_3 = 24} \qquad Eq4$$

Apply $(Eq3) + 2(Eq1)$:

$$2I_2 - 5I_3 + 12I_1 - 2I_2 = 16$$
$$\mathbf{12I_1 - 5I_3 = 16} \qquad Eq5$$

Apply $12(Eq4) - 17(Eq5)$:

$$204I_1 - 24I_3 - 204I_1 + 85I_3 = 288 - 272$$
$$61I_3 = 16$$
$$\mathbf{I_3 = 0.262A} \qquad Eq6$$

Use $(Eq6)$ in $(Eq3)$

$$\mathbf{I_2 = \frac{5I_3}{2} = \frac{5(0.262)}{2} = 0.656A} \qquad Eq7$$

Use $(Eq7)$ in $(Eq1)$

$$\mathbf{I_1 = \frac{8 + I_2}{6} = \frac{8 + 0.656}{6} = 1.443A} \qquad Eq8$$

Learning Outcomes

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  - Power & Energy
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  - Solution of a simple electrical circuit using just Ohm's and Kirchhoff's Laws
- Electrical circuits
  - **Series** & **Parallel**
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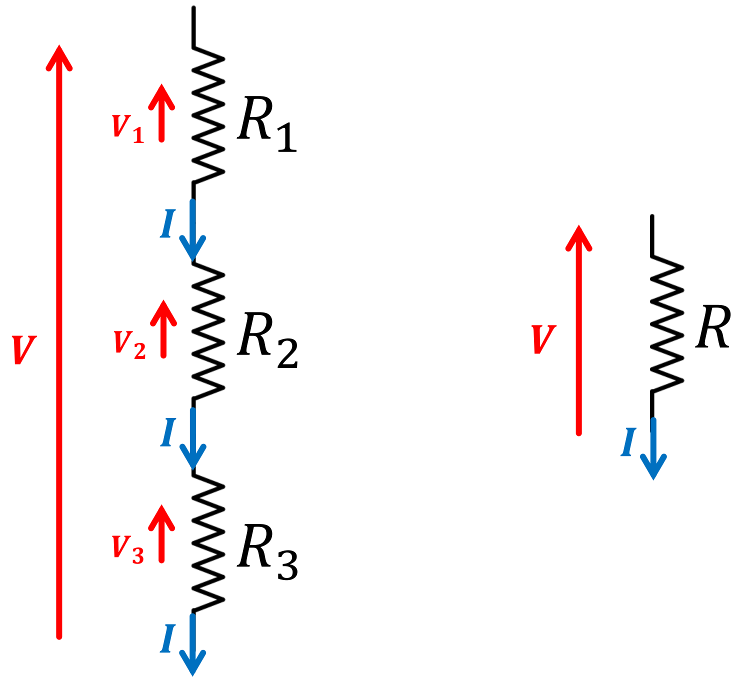
- Further Reading

Series Circuit

When two (or more) elements are connected together head-to-toe



Same current flows through each element

$$I = I_1 = I_2 = I_3$$

Voltage gets split between elements

$$V = V_1 + V_2 + V_3$$

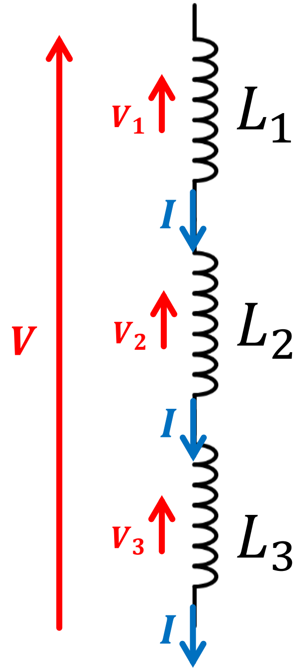
More resistors in series, the harder it is for voltage source to push the current through

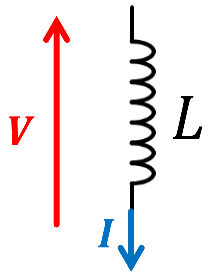
Resistance value adds up

$$R = R_1 + R_2 + R_3$$

Series Circuit

When two (or more) elements are connected together head-to-toe





Same current flows through each element

$$I = I_1 = I_2 = I_3$$

Voltage gets split between elements

$$V = V_1 + V_2 + V_3$$

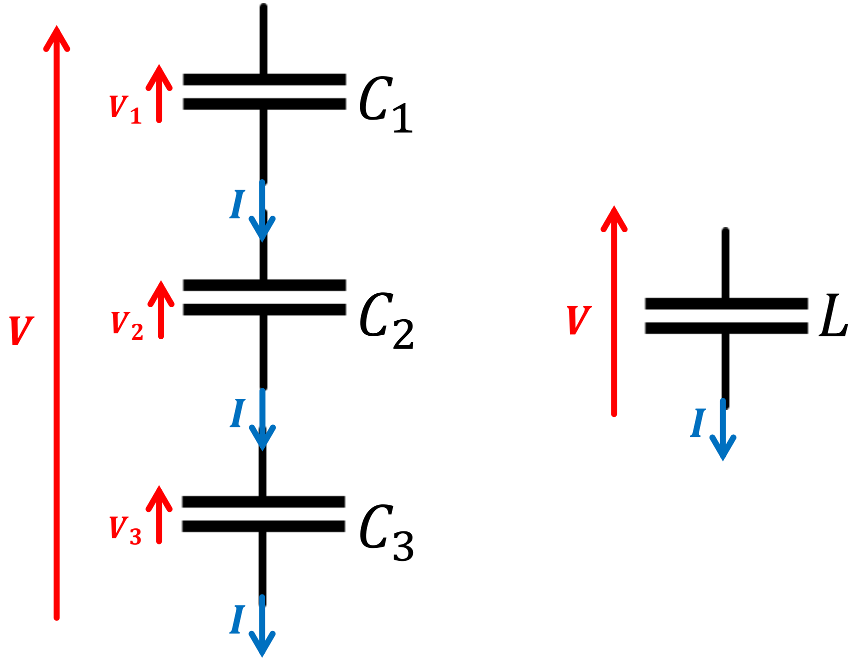
More inductors in series, the harder it is for current to change rapidly

Inductance value adds up

$$L = L_1 + L_2 + L_3$$

Series Circuit

When two (or more) elements are connected together head-to-toe



Same current flows through each element

$$I = I_1 = I_2 = I_3$$

Voltage gets split between elements

$$V = V_1 + V_2 + V_3$$

More capacitors in series, the easier it is for voltage to change rapidly

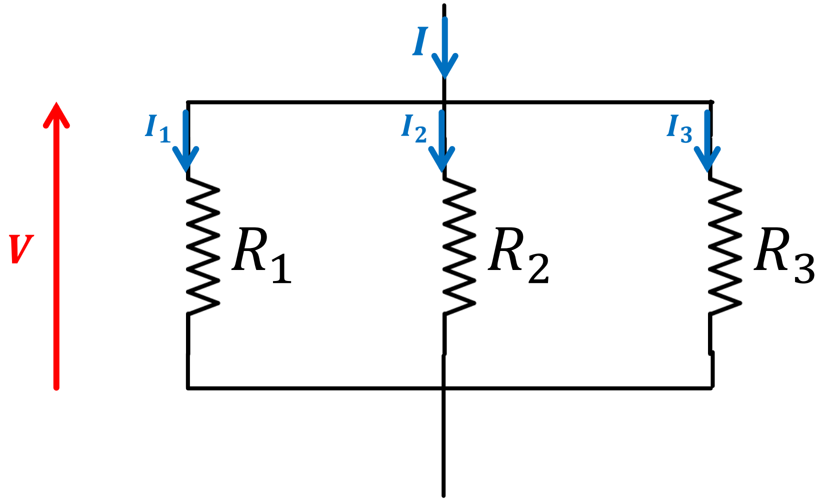
Apply KVL: in series, voltage gets divided, so each capacitor needs to oppose change of only part of the total voltage change

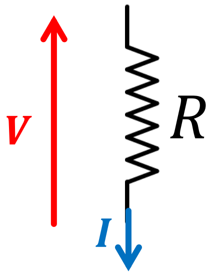
Reciprocal of capacitance value adds up

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Parallel Circuit

When two (or more) elements are connected together head-to-head





Same current flows through each element

$$I = I_1 + I_2 + I_3$$

Voltage gets split between elements

$$V = V_1 = V_2 = V_3$$

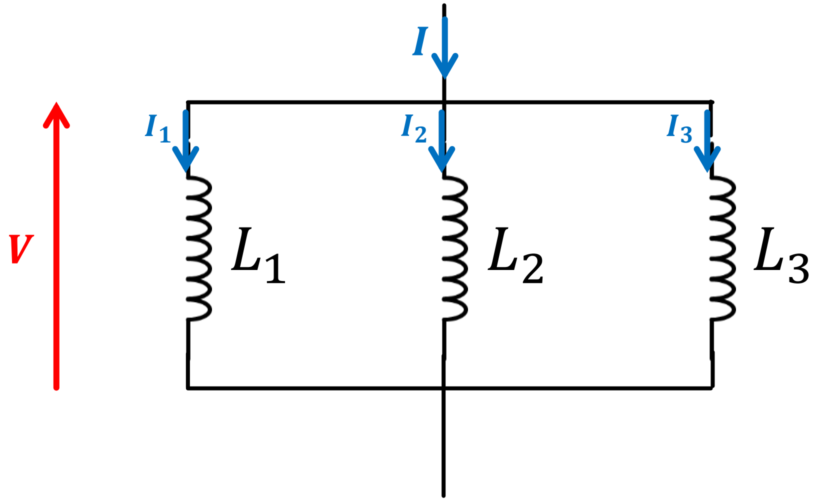
More resistors in parallel, more "effective paths" for electrons to pass through

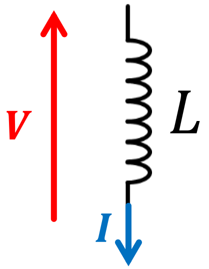
Reciprocal of resistance value (conductance) adds up

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Parallel Circuit

When two (or more) elements are connected together head-to-head





Same current flows through each element

$$I = I_1 + I_2 + I_3$$

Voltage gets split between elements

$$V = V_1 = V_2 = V_3$$

More inductors in parallel, easier it is for current to change rapidly

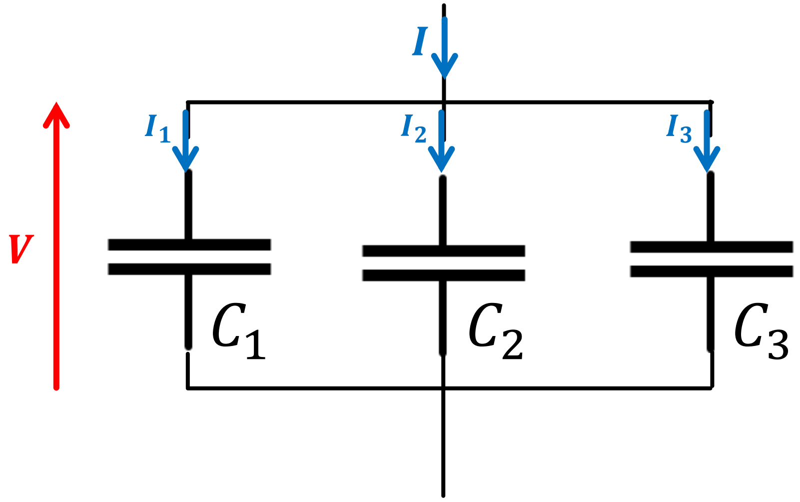
Apply KCL: in parallel, current gets divided, so each inductor needs to oppose change of only part of the total current change

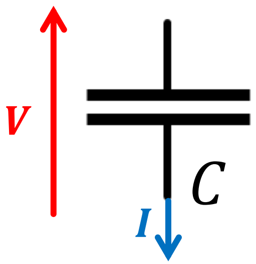
Reciprocal of inductance value adds up

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Parallel Circuit

When two (or more) elements are connected together head-to-head





Same current flows through each element

$$I = I_1 + I_2 + I_3$$

Voltage gets split between elements

$$V = V_1 = V_2 = V_3$$

More capacitors in parallel, harder it is for voltage to change rapidly

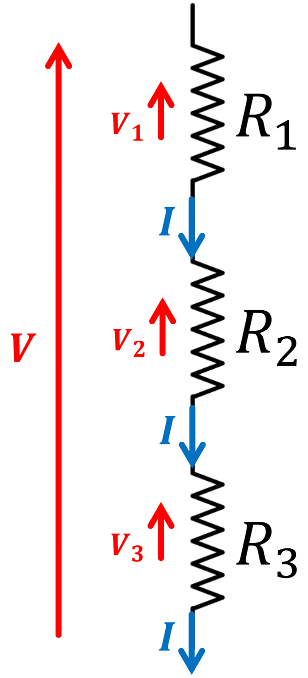
Reciprocal of inductance value adds up

$$C = C_1 + C_2 + C_3$$

Types of Circuit

Series

When two (or more) elements are connected together head-to-toe

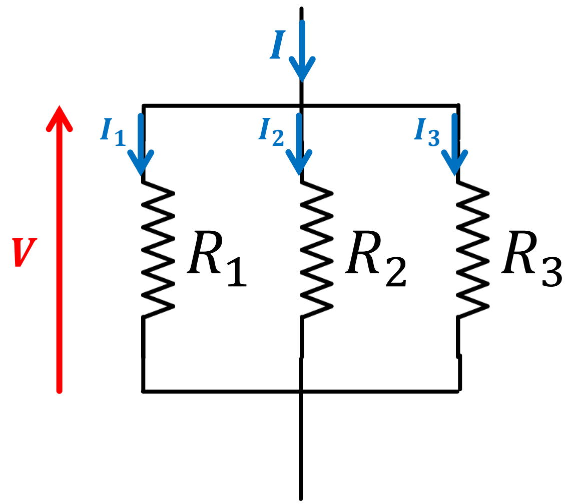


Same current flows through each element

Voltage gets split: $V = V_1 + V_2 + V_3$

Parallel

When two (or more) elements are connected head-to-head and toe-to-toe

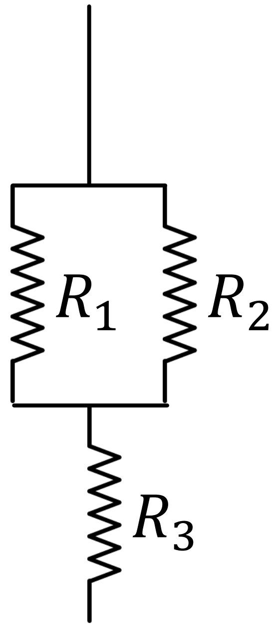


Same voltage across each element

Current gets split: $I = I_1 + I_2 + I_3$

Series-Parallel

Combination of the both

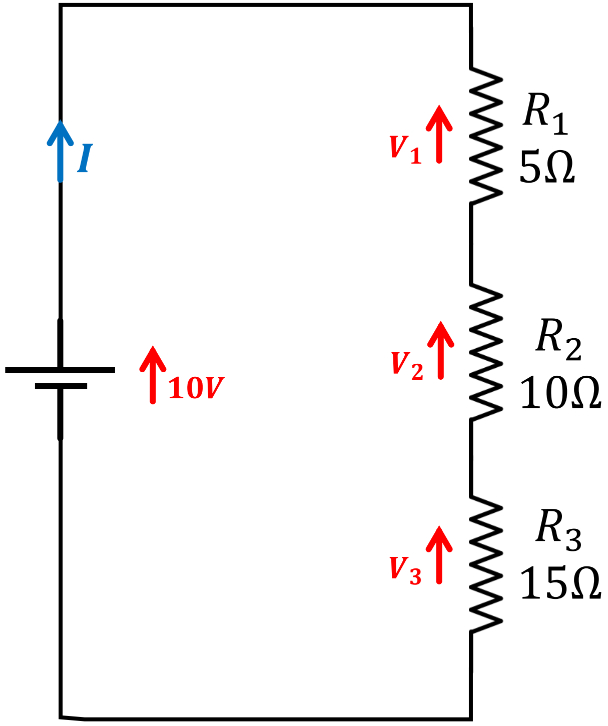


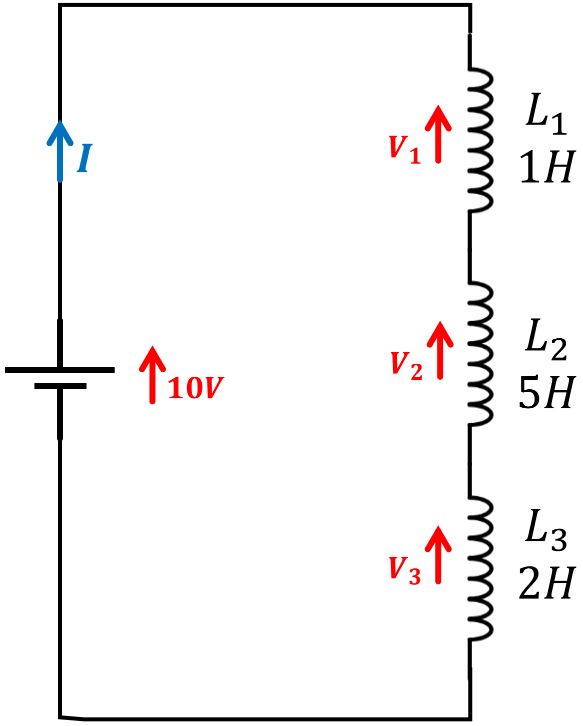
Break the circuit up into series and parallel and solve individually

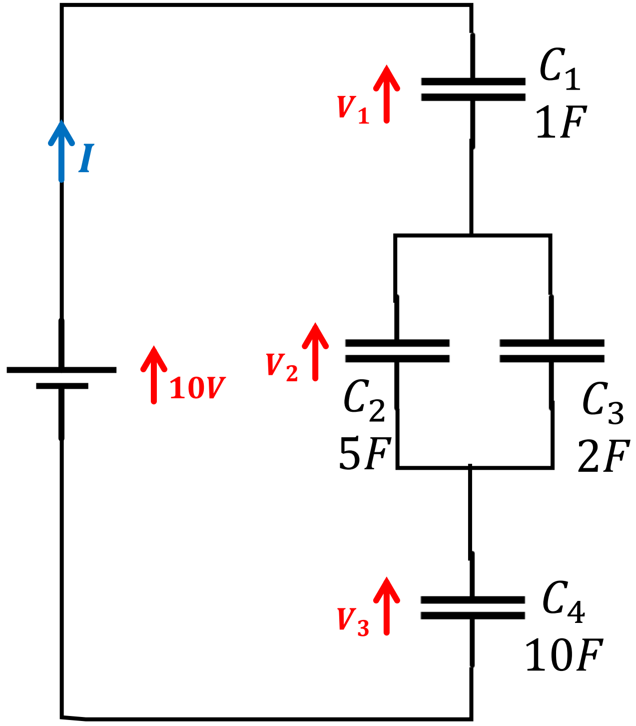
Types of Circuit

	Resistor	Inductor	Capacitor
V-I relation	$V = IR$	$V = L\frac{dI}{dt}$	$I = C\frac{dV}{dt}$
Power	$P = VI = \frac{V^2}{R} = I^2R$	$P = VI$	$P = VI$
Energy Stored	No energy stored	$E = \frac{1}{2}CV^2$	$E = \frac{1}{2}LI^2$
Series	$R = \sum R_i$	$L = \sum L_i$	$\frac{1}{C} = \sum \frac{1}{C_i}$
Parallel	$\frac{1}{R} = \sum \frac{1}{R_i}$	$\frac{1}{L} = \sum \frac{1}{L_i}$	$C = \sum C_i$

Can you prove these formulae using Kirchhoff's and Ohm's Laws?







- What is the values of V_1, V_2, V_3?
- What is the value of I?

- Prove that the set of three inductors can be equivalently replaced with an inductor with inductance of $8H$

- Prove that the set of four capacitors can be equivalently replaced with a capacitor with capacitance of $0.805F$

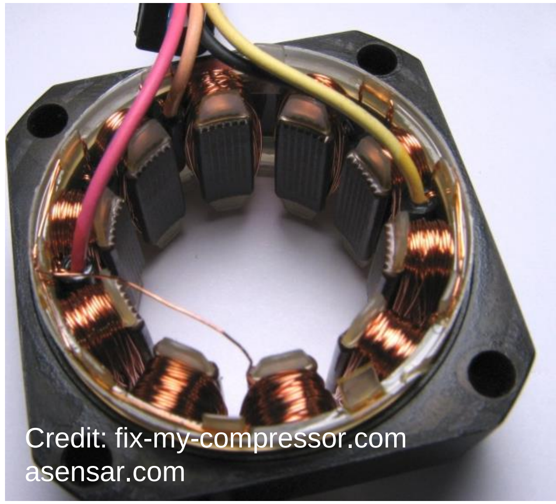
Summary

- **Basic Concepts (& Recap)**
  - Charge, current, voltage
  - Ohm's/Kirchhoff's Laws
  - Power & Energy
  - Measurement of Voltage & Current
  - Electrical symbols & notations
  - Solution of a simple electrical circuit using just Ohm's and Kirchhoff's Laws
- Electrical circuits
  - **Series** & **Parallel**
  - **Combination** of series & parallel
  - **Example circuits** (to be discussed in upcoming seminar)
- Further Reading

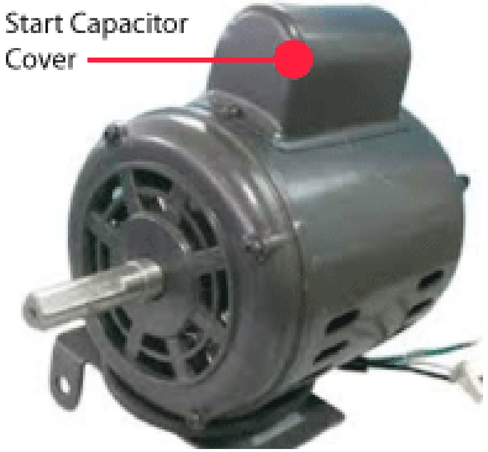
Further Reading

Inductors & Capacitors – why bother?

- Almost all motors, actuators etc. have (unwanted) inductance due to their coils of wire (windings)
- Capacitors are widely used for filtering and smoothing signals, and creating phase shifts e.g. to start some kinds of motor

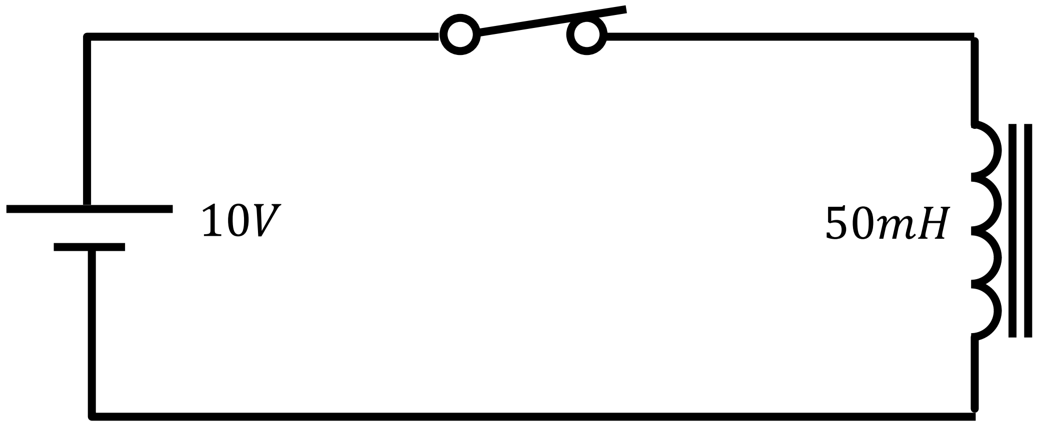


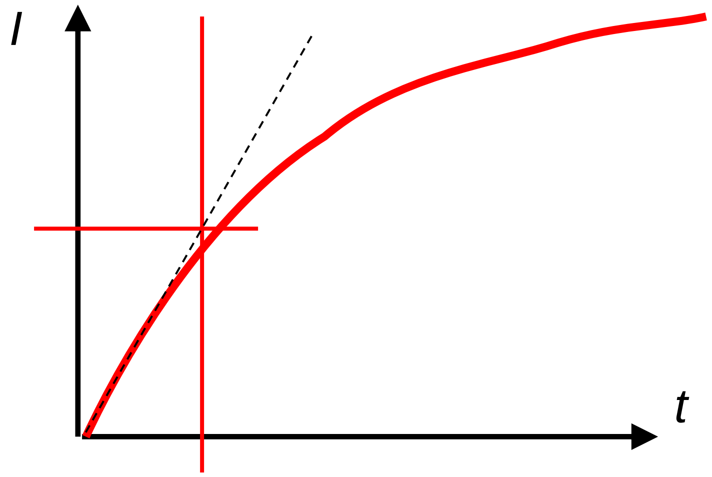
Credit: fix-my-compressor.com
asensar.com



Example of Inductor calculation

- A solenoid actuator has an inductance of $50mH$. $0.01s$ after connecting it to a $10V$ dc supply, what value of current is flowing? Ignore the resistance of the actuator.





Example of Inductor calculation

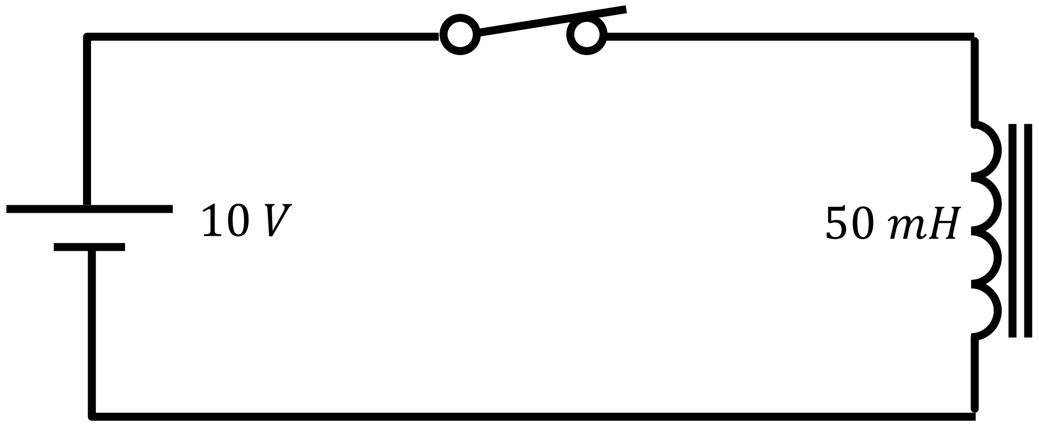
$$V = L\frac{dI}{dt}$$

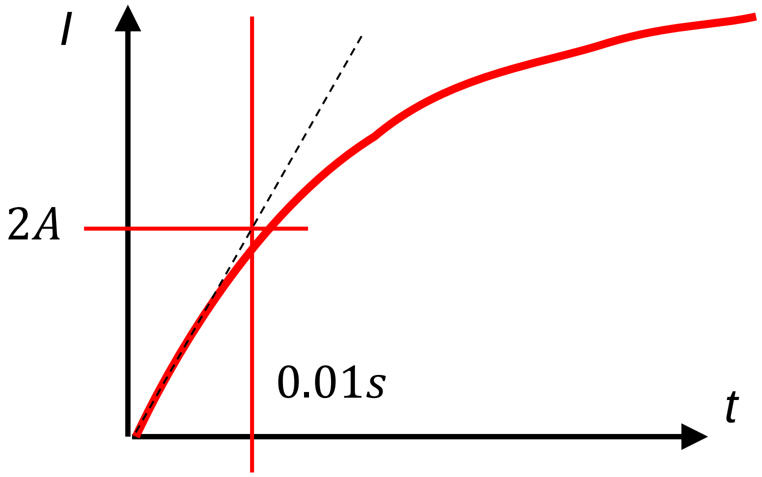
$$\frac{dI}{dt} = \frac{V}{L} = \frac{10}{50 \times 10^{-3}}$$

$$= 200\ A/s$$

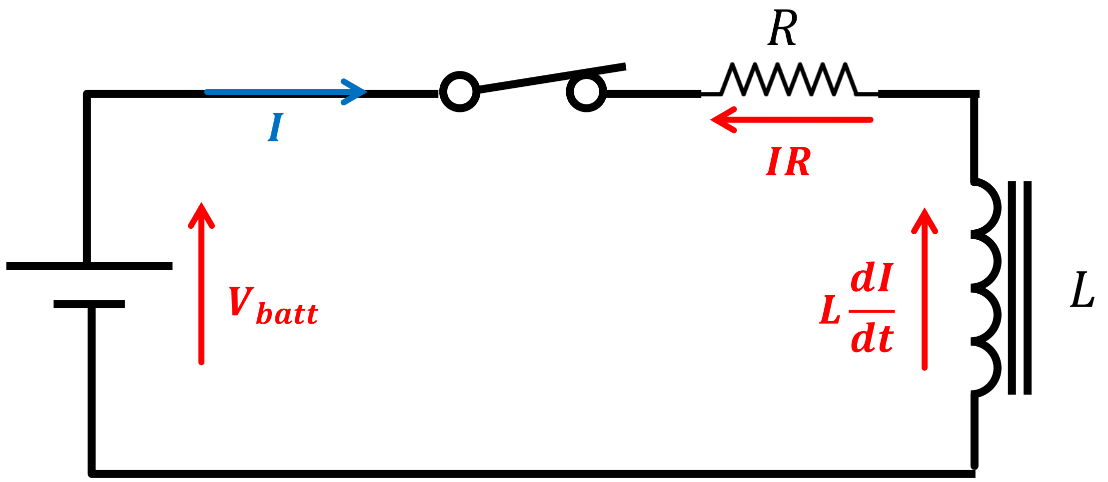
So,

$$\boldsymbol{\Delta I} \approx \left(\frac{dI}{dt}\right)\Delta t = 200 \times 0.01 = \boldsymbol{2A}$$





Example of Inductor calculation – more realistic problem

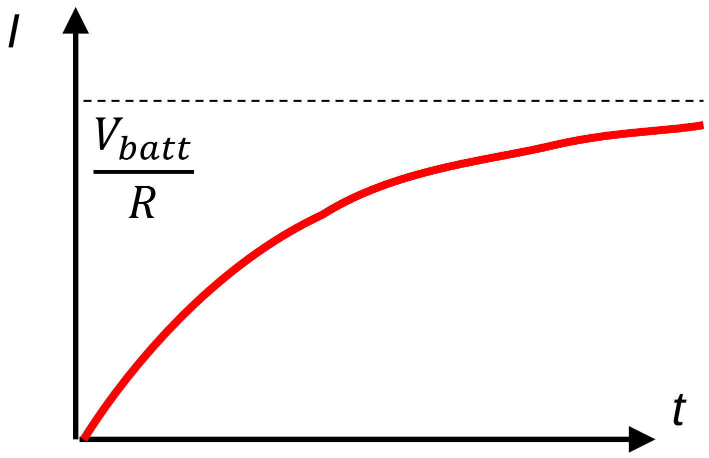


$$V_{batt} = IR + L\frac{dI}{dt}$$

This is a differential equation, so we need to integrate this

$$V_{batt}\,dt = IRdt + LdI$$

$$V_{batt}\int dt = R\int I.dt + L\int dI$$

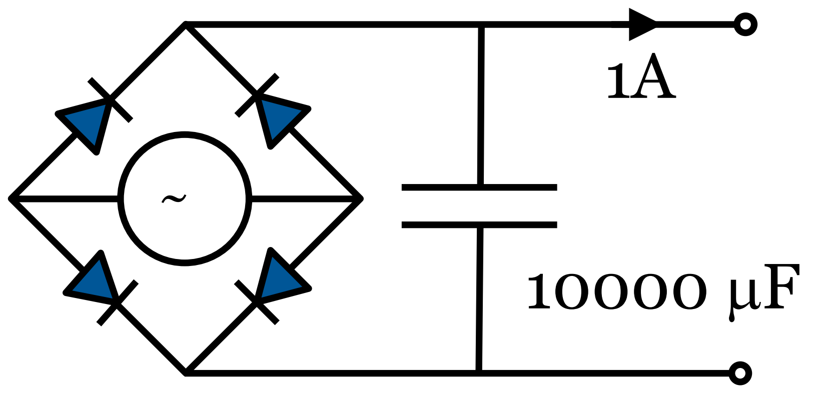


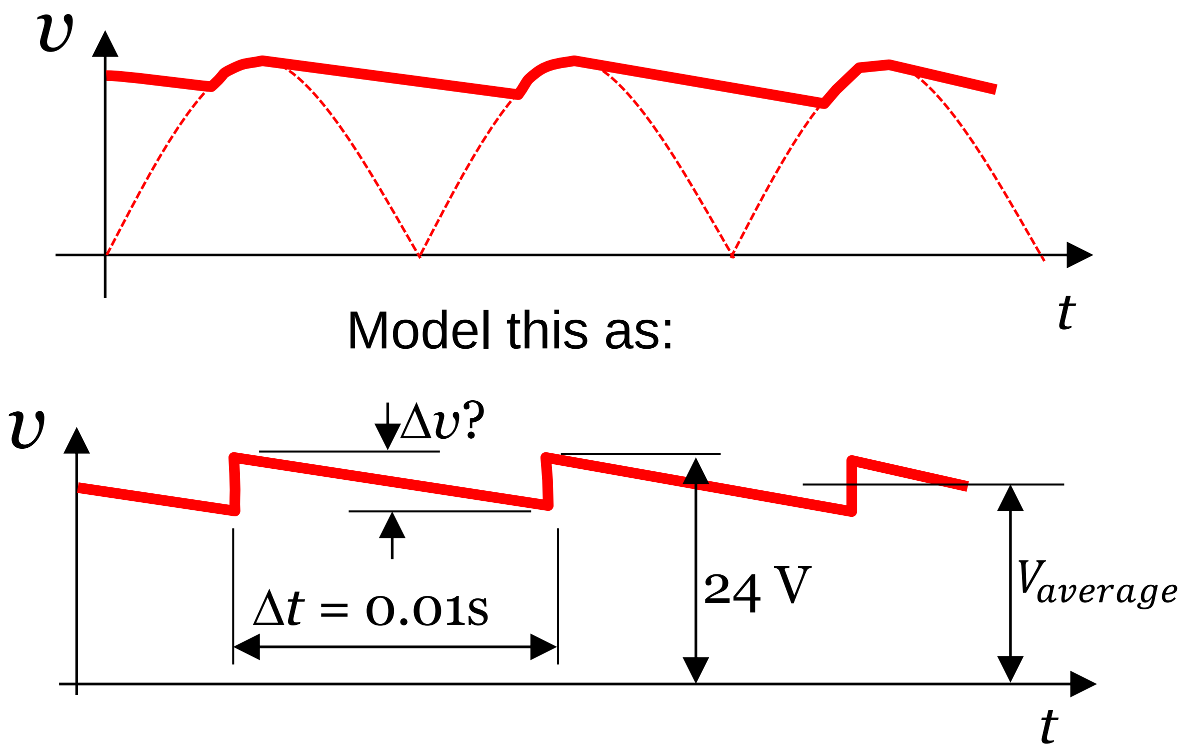
After some complex calculus

$$\boldsymbol{I(t) = \frac{V_{batt}}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

Example of Capacitor calculation

- A $10mF$ capacitor is used to smooth the output of a rectified $50Hz$ power supply.
- Effectively, the capacitor is charged by a voltage peak to $24V$ every $0.01s$
- $1A$ is drawn from the power supply
- By what amount does the output voltage drop between charging peaks?
- What is mean voltage?
- Specifically, what is the voltage change Δv in $0.01s$?
- What is $V_{average}$?





Example of Capacitor calculation

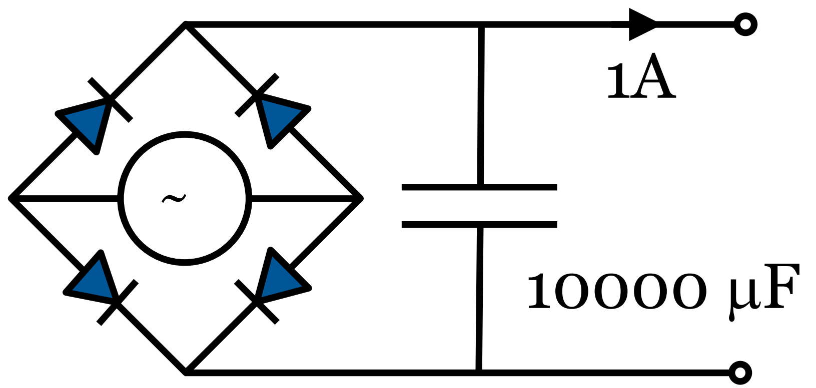
$$I = C\frac{dV}{dt} = -1\text{A}$$

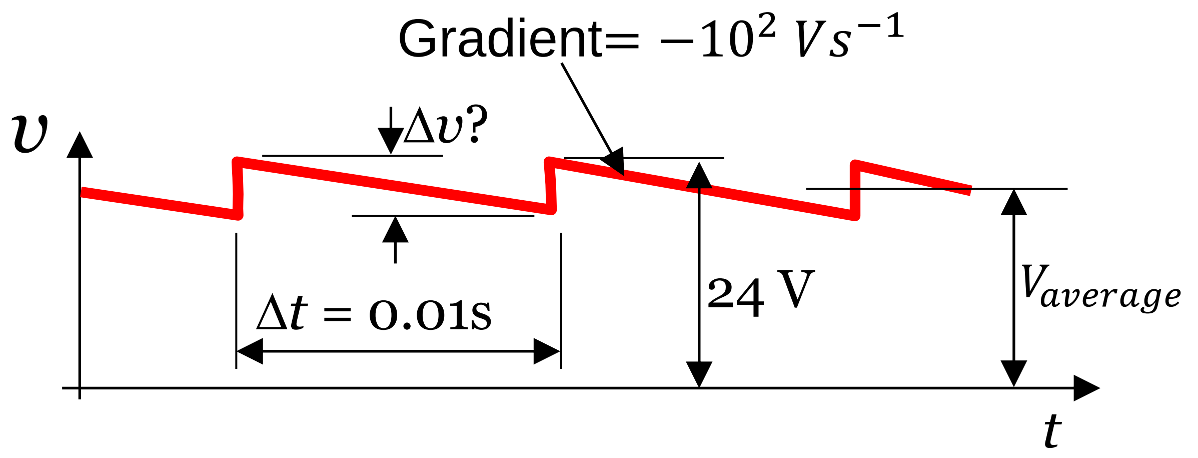
(current flowing out of cap so − ve)

$$\text{so } \frac{dV}{dt} = \frac{I}{C} = \frac{-1}{10000 \times 10^{-6}} = -10^2 \text{ Vs}^{-1}$$

$$\text{So } \Delta v = \frac{dV}{dt}\Delta t = -10^2 \times 0.01 = -1\text{V}$$

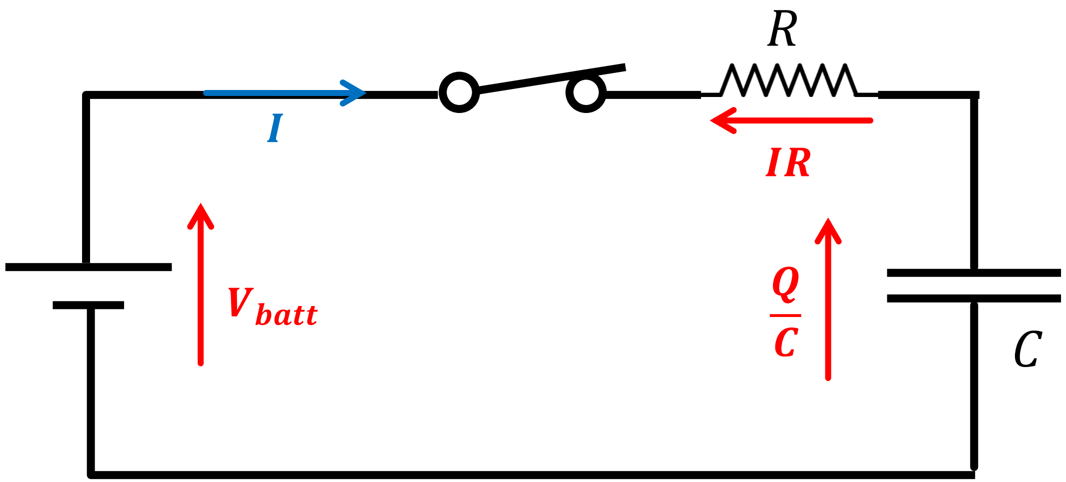
$$\text{Mean voltage is therefore } \frac{24 + (24-1)}{2} = \mathbf{23.5V}$$





Another example of Capacitor calculation

- A capacitor of value $1000\mu F$ is connected to a $12V$ battery via a 10000Ω resistor. At what rate does the capacitor voltage increase initially?



Another example of Capacitor calculation

Initially: voltage across capacitor = 0

So

$$V_{batt} - IR - 0 = 0$$

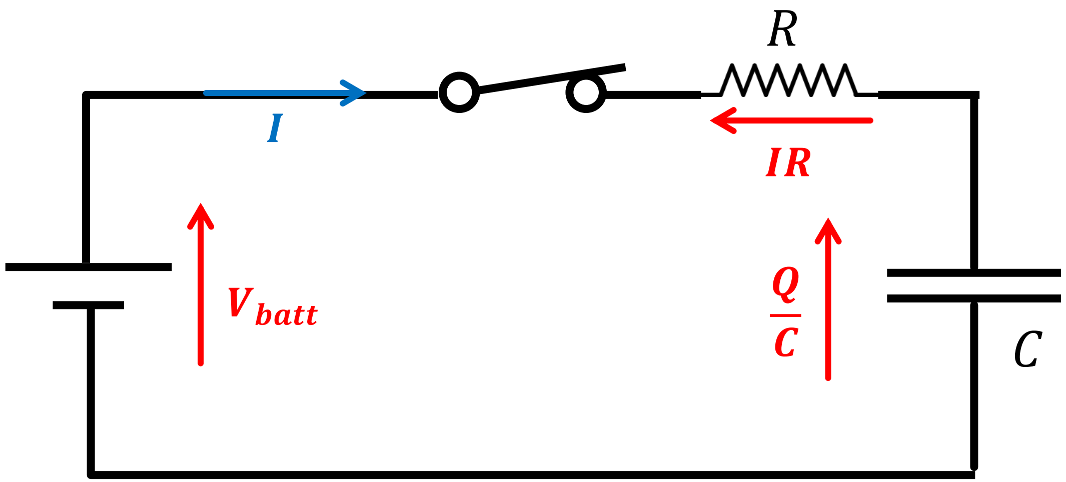
$$I = \frac{V_{batt}}{R}$$

But

$$I = \frac{dQ}{dt} = C\frac{dV_{cap}}{dt}$$

So

$$\frac{dV_{cap}}{dt} = \frac{V_{batt}}{RC} = \frac{12}{10000 \times 1000 \times 10^{-6}} = 1.2\,Vs^{-1}$$



Another example of Capacitor calculation