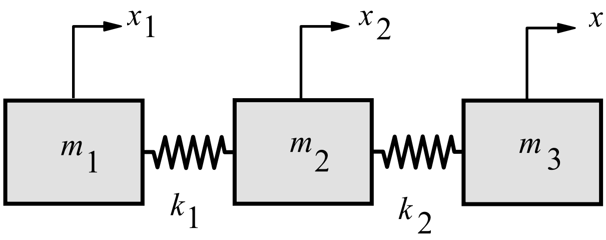
Department of Mechanical, Materials & Manufacturing Engineering

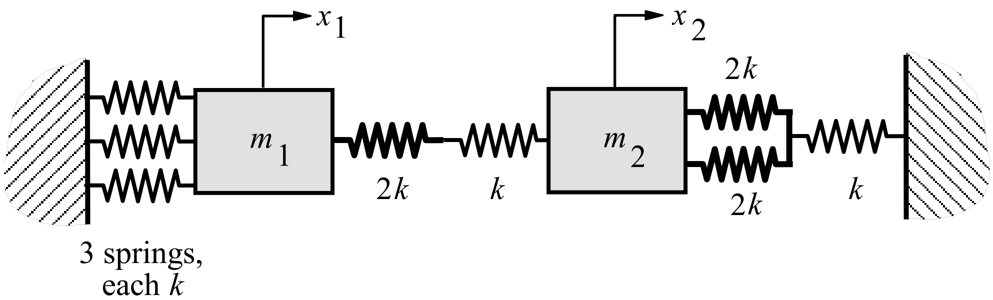
DYNAMICS (VIBRATION)

SHEET 2 : MULTI-DEGREE-OF-FREEDOM SYSTEMS

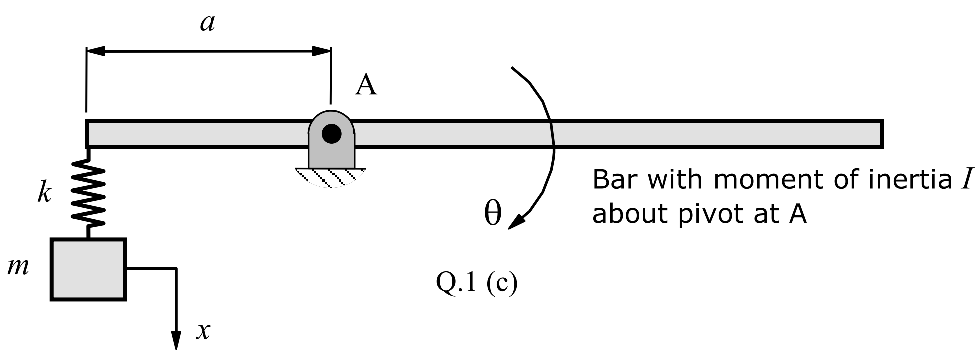
1 Derive the equations of motion for each of the following systems. Assume that all displacements and angles are small.



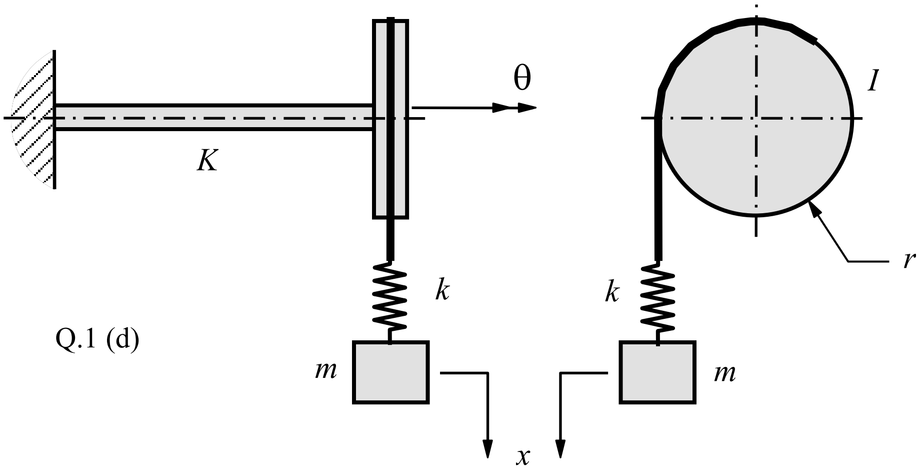
Q.1 (a)



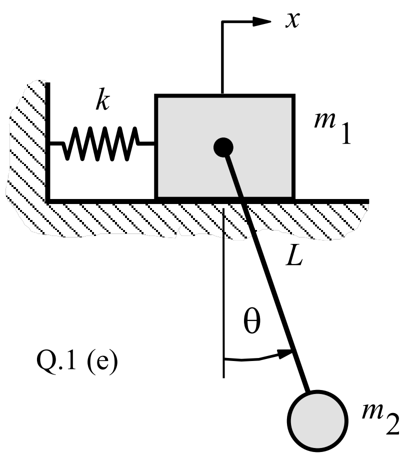
Q.1 (b)



Q.1 (c)



Q.1 (d)



Q.1 (e)

(a) $$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

(b) $$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + k \begin{bmatrix} \frac{11}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{22}{15} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

(c) $$\begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + k \begin{bmatrix} a^2 & a \\ a & 1 \end{bmatrix} \begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

(d) $$\begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} K + k r^2 & -k r \\ -k r & k \end{bmatrix} \begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

(e) $$\begin{bmatrix} m_1 & 0 \\ m_2 & m_2 L \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k & -m_2 g \\ 0 & m_2 g \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

2. A single-axle caravan has the following data: body mass 350 kg; unsprung mass 70 kg; two suspension springs, each of stiffness 20 kN/m; two tyres, each of stiffness 200 kN/m.

(a) By assuming that the tyres are rigid in comparison with the suspension springs, use a single degree-of-freedom dynamic model and estimate the lowest natural frequency of the caravan.

(b) Find the natural frequencies and the corresponding mode shapes for a two-degree-of-freedom model.

1.70 Hz; *Mode 1* *1.62 Hz,* $X_{AXLE} : X_{BODY} = 0.0924 : 1$
Mode 2 *12.6 Hz,* $X_{AXLE} : X_{BODY} = -54.1 : 1$

3. Find the natural frequencies and the corresponding mode shapes for the system in Q.1 (a) when k_1 = 10 kN/m, k_2 = 30 kN/m, $m_1 = m_2$ = 5 kg and m_3 = 10 kg.

Mode	Frequency (Hz)	Mode shape		
		X_1	X_2	X_3
1	0.00	1.0	1.0	1.0
2	7.54	-8.77	1.0	3.89
3	16.5	-0.228	1.0	-0.386