Strain Energy Methods

Lecture 1 – Introduction & Definition

Department of Mechanical, Materials & Manufacturing Engineering

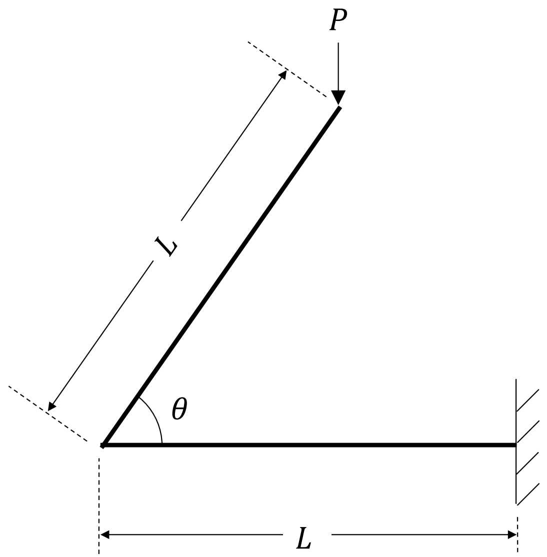
MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

Strain Energy Methods

Introduction

We have seen in the Deflection of Beams topic how deflections and slopes of a beam can be determined by solving the differential equation of the elastic line, i.e., Macaulay's method. However, this method is not appropriate for more complex shaped structures or bodies where deflections occur.



Here we introduce the concept of **strain energy**, which will enable us to calculate deflections of such complex structures.

When a material is subject to loading, strain energy is stored within it. An Italian railway engineer, named Castigliano, derived a theorem and procedure for using this strain energy to determine deflections in structures or bodies. Castigliano's theorem is a powerful and flexible method for solving deflection problems.

Strain Energy Methods

Learning Outcomes

1. Know the basic concept of strain energy stored in a material body under loading (knowledge);
2. Be able to calculate strain energy in an elastic body/structure arising from various types of loading, including tension/compression, bending and torsion (application);
3. Be able to apply Castigliano's theorem for linear elastic bodies to enable the deflection or rotation of a body at a point to be calculated from strain energy expressions (application).

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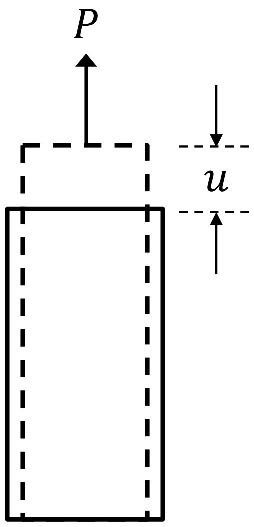
Strain Energy Definition

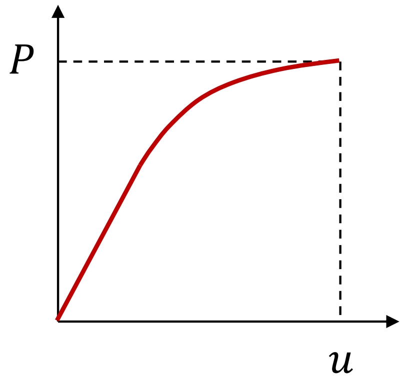
Axial Loading

The strain energy in a material body is equal to the work done on the body by the applied loads. Thus, if an elastic-plastic material body is subjected to a single axial load, P, as shown in part (a) of the figure below, causing a displacement, u, at the load application point, according to the behaviour shown in part (b) of the figure, then the strain energy, U, is given as:

$$U = \int_0^u P \mathrm{d}u$$



(a)



(b)

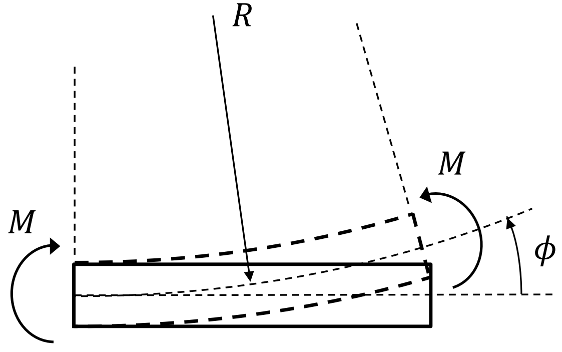
Strain Energy Definition

Bending and Torsion

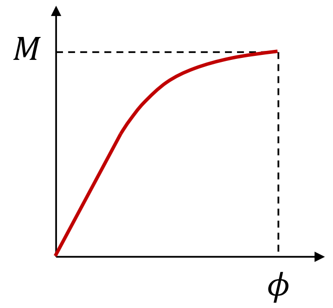
Similar expressions can be derived for bending of a beam and torsion of a bar, according to the figures below, as:

Bending

$$U = \int_0^{\phi} M \mathrm{d}\phi$$



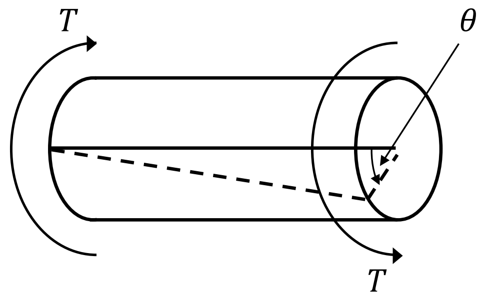
(a)



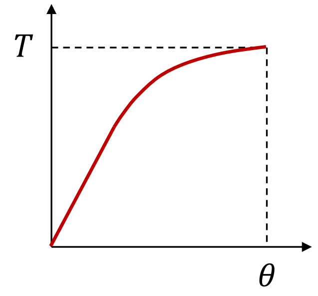
(b)

Torsion

$$U = \int_0^{\theta} T \mathrm{d}\theta$$



(a)



(b)

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